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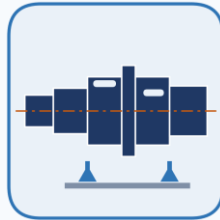
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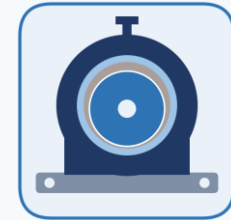
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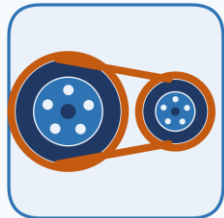
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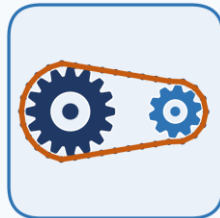
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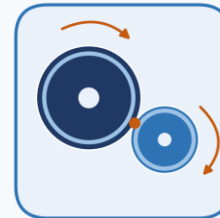
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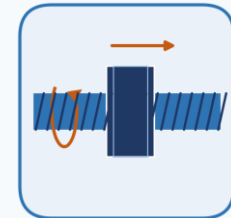
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AEV. INVOLUTE GEARS

AEV-T. STRUCTURE, CONSTRUCTION AND MODELING OF INVOLUTE GEARS

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AEV-T.1 STRUCTURE AND CONSTRUCTION OF INVOLUTE GEARS

AEV-T.1.1 DEFINITION

The gear is the mechanism composed of two gear wheels that transmits *rotational motion* and *torque* through meshing (the direct and continuous contact of the conjugate tooth profiles).

The gear wheel is a machine element with teeth distributed at equal angular spacing around the periphery with the angular pitch $\tau = 2\pi/z$ (fig. AEV-T.1.1,b,c).

Pinion – the gear wheel of a gear with the minimum number of teeth.

A wheel tooth is a protrusion on the circumference of a wheel with a symmetrical structure and active flanks suitable for engagement.

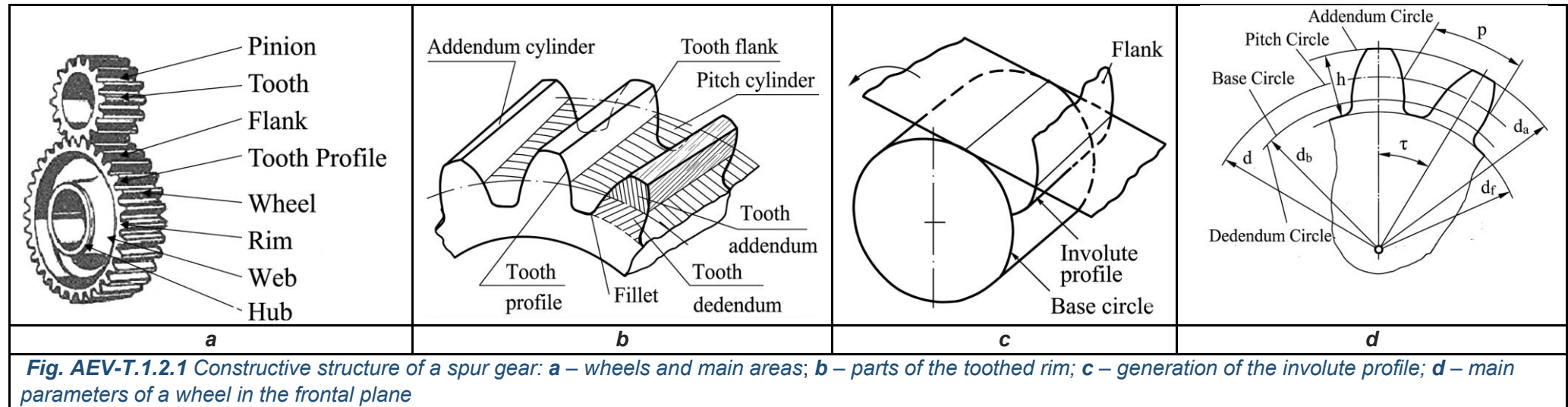
Gearing is the continuous process of successive and continuous contact of the mating wheels of a gear, ensuring uninterrupted transmission of motion.

The tooth flank (fig. AEV-T.1.2.1,a,b) is the active surface of a tooth that contributes to the meshing process.

The tooth profile (fig. AEV-T.1.2.1,b) is the curve defining the shape of the active flank; usually the involute profile is used (fer. [AEV-T.1](#)).



AEV-T.1.2 CONSTRUCTION STRUCTURE



AEV-T.1.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Main areas of use: gearboxes, speed reducers, speed multipliers, differential transmissions, conveyors, elevators, technological equipment, oil installations, etc.

Tab. AEV-T.1.3.1 Advantages and disadvantages of gear transmissions [Jula, 1989; Chişiu, 1981]

Advantages *	Disadvantages *
- increased load-bearing capacity (reduced dimensions) - high efficiency - high durability - possibility of use for wide ranges of powers, speeds, transmission ratios - operational safety - constant transmission ratio	- high execution and assembly precision - complex technologies (high costs) - noise and vibrations during operation - discrete transmission ratios (the numbers of teeth on the wheels are integers)
* Compared to other types of transmissions (through flexible elements, through friction, etc.)	

AEV-T.1.4 CLASSIFICATION

Tab. AEV-T.4.1 Classification of involute gears [Moldovean, 2001]

criterion	Type	Scheme
Relative position of the wheel axles (see fig. AEV-T.1.7.1)	Gears with parallel axes (fixed or movable; $d > 0$, $\delta = 0$, $\theta \in [0, \pi/2]$)	Fig. AEV-T.1.4.1,a, b, c, d
	Rack and pinion gears ($d \rightarrow \infty$, $\delta = 0$, $\theta \in [0, \pi/2]$)	Fig. AEV-T.1.4.1,e

	Gears with concurrent axes (any, orthogonal; $d = 0$, $\delta \in [0, \pi/2]$, $\theta \in [0, \pi/2]$)	Fig. AEV-T.1.4.1,f, g, h, i
	Cross-axis gears (any, orthogonal; $d > 0$, $\delta \in [0, \pi/2]$, $\theta \in [0, \pi/2]$)	Fig. AEV-T.1.4.1,i, j, k
Tooth shape	Straight-tooth gears (with spur or helical toothing)	Fig. AEV-T.1.4.1,a-i, k
	Gears with curved teeth	Fig. AEV-T.1.4.1,i, j
Wheel shape	Cylindrical gears	Fig. AEV-T.1.4.1, a, b, c, d, e
	Bevel gears	Fig. AEV-T.1.4.1, g, h, i
	Conical-cylindrical gears	Fig. AEV-T.1.4.1,f
	Hyperboloidal gears: helical, worm, hypoid	Fig. AEV-T.1.4.1,j,k,l
Tooth flank profile	Involute (fer. AEV-T.1)	Fig. AEV-T.1.4.1,a-l
	Cycloidal, epicycloidal	-

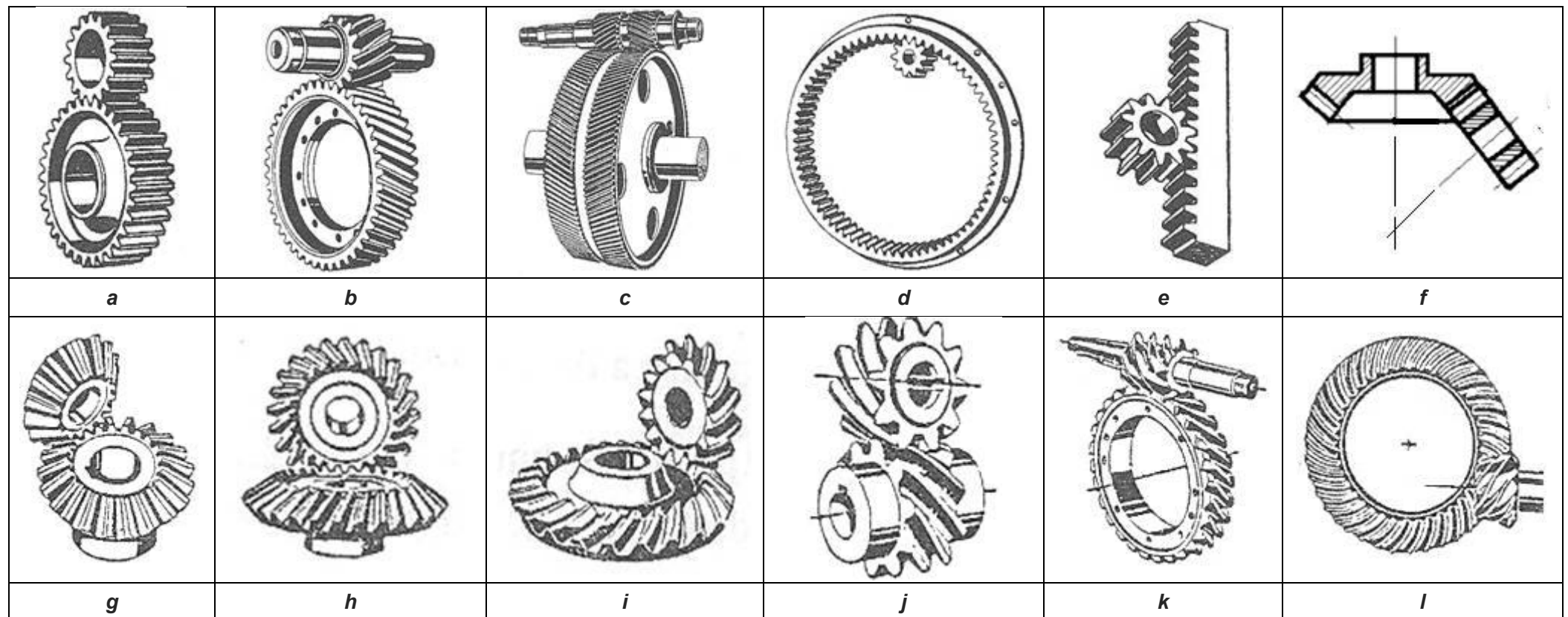


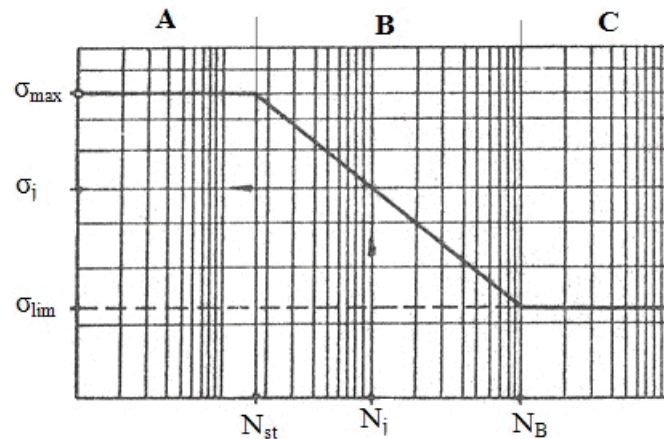
Fig. AEV-T.1.4.1 Types of gears: **a** – external cylindrical with straight teeth; **b** – external cylindrical with inclined teeth; **c** – external cylindrical with V-shaped teeth; **d** – internal cylindrical; **e** – rack-and-pinion; **f** – conical-cylindrical; **g** – orthogonal conical with straight teeth; **h** – orthogonal conical with inclined teeth; **i** – orthogonal conical with curved teeth; **j** – helical; **k** – worm; **l** – hypoid

AEV-T.1.5 MATERIALS AND TECHNOLOGIES

AEV-T.1.5.1 Materials and heat treatments

Tab. AEV-T.1.5.1.1 Material groups and technological flows for gears [Cheša, 1984]

Material group	Common brands	Technological flows								
Carbon steels for heat treatment improvement, SR EN 10083	C45, C50, C55, C60	Heat treatment for property improvement: quenching + high tempering, 150-350 HB	→	Tooth processing by milling or gear shaping	→	Tooth finishing (e.g. shaving, for cylindrical gears)	or	Surface hardening heat treatment, 350-500 HB, surface (3...8 mm)	or	Thermochemical nitriding treatment, 40-55 HRC, on the surface (0.2...0.8 mm)
Alloyed heat-treatable steels, STAS 791	33MoCr11, 40Cr10, 41 CrNi 12, 42 MoCr 11									
Carburizing quality carbon steels, STAS 880	C15, C20	Tooth processing by milling or gear shaping	→	Case hardening heat treatment: carburizing + quenching + low tempering, 45-65 HRC (surface, 0.8...1.2 mm)	→	Tooth finishing by grinding (mandatory)				
Alloyed case-hardening steels, STAS 791	15 Cr 9, 18 MnCr11, 20TiMnCr12,, 18CrNi20, 20MoNi35, 17MoCrNi14									
Other materials	Malleable cast iron (Fmp 700-02, STAS 569), with nodular graphite (Fgn 700-2, STAS 6071) or antifriction (STAS 6707); bronzes (CuSn10; CuSn6Zn4Pb4 -STAS 197/2), plastic materials (bakelite, textolite, polyamides)									

Tab. AEV-T.1.5.1.2 Specific strength characteristics of gear materials. Operating modes**Fig. AEV-T.1.5.1.1** Wöhler general fatigue curve (for contact and bending) and operating modes

The **limiting stress**, σ_{lim} (σ_{Hlim} for contact or σ_{Flim} for bending) is the maximum stress (at contact or at bending) at which after a number of basic operating cycles N_B (N_{BH} , for contact; N_{BF} , for bending) the wear/fatigue damage of the active surfaces by pitting or tooth breakage occurs at the limit. The values of the limiting stresses (contact and bending) determined experimentally under laboratory conditions can be found in the specialized literature in tabular or graphical form (fer. [AEV-M.1](#)).

The operation of a gear wheel depending on the number of stress cycles (contact or bending) N_j can fall into one of the following regimes (fig. AEV-T.1.5.1.1):

A - static stresses ($N_j \leq N_{st}$), material fatigue does not occur due to the low number of stress cycles ($N_{st} < 1000$).

B - limited durability ($N_{st} < N_j \leq N_B$), fatigue failures of the material occur after N_j cycles of stress less than the number of cycles of the basic stress N_B (approx. $10^{6...8}$). From the equation of the fatigue curve,

$\sigma_j^m N_j = \sigma_{lim}^m N_B$, the value of the design stress is determined, $\sigma_j = \sqrt[m]{N_B/N_j} \sigma_{lim}$, with m the durability exponent (slope of the inclined branch of the fatigue curve).

C - unlimited durability ($N_j > N_B$), the fatigue phenomenon does not occur for (theoretically) a number of load cycles greater than the basic one

AEV-T.1.5.2 Processing technologies

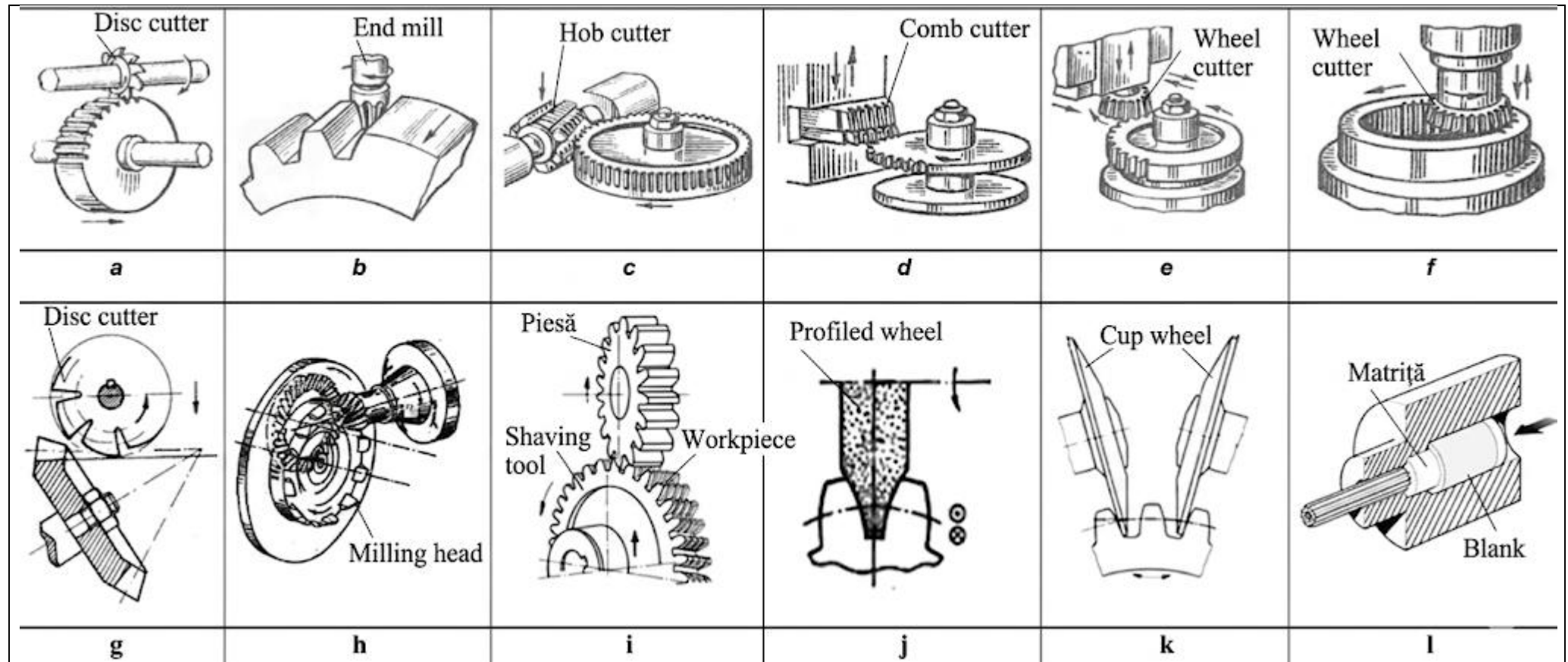


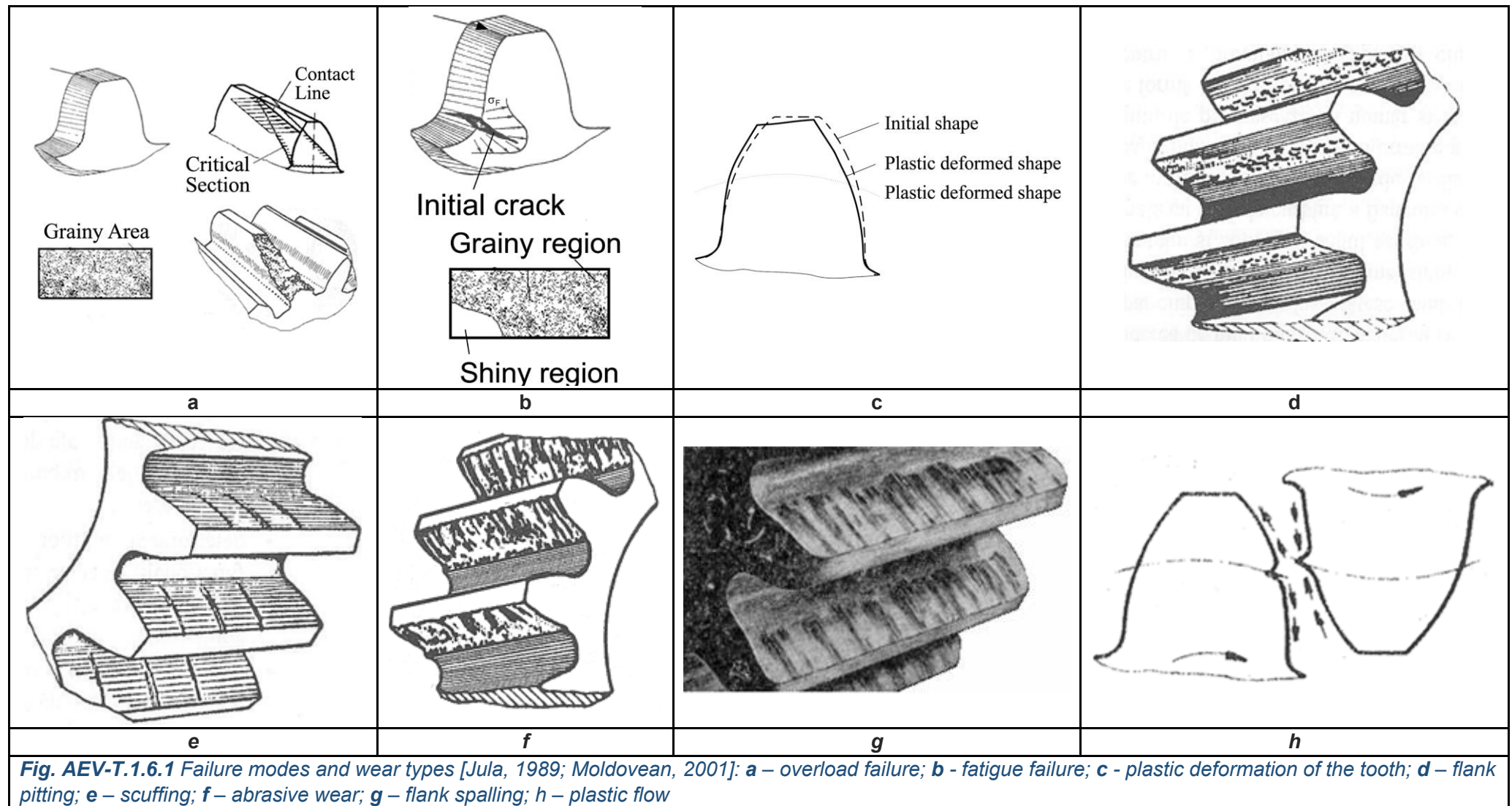
Fig. AEV-T.1.5.2.1 Tooth processing processes [Moldovean, 2001]: **a** – milling by copying with a disk cutter; **b** – milling by copying with a finger cutter; **c** – milling by rolling with a worm cutter; **d** – external gear tooth shaping by rolling with a comb cutter; **e** – external gear tooth shaping by rolling with a wheel cutter; **f** – internal gear tooth shaping by rolling with a wheel cutter; **g** – conical tooth milling by copying with a disk cutter; **h** – curved conical tooth milling by rolling; **i** – tooth finishing by shaving; **j** – tooth grinding (finishing) by form grinding; **k** – tooth grinding (finishing) by generating; **l** – plastic deformation (extrusion) of teeth

AEV-T.1.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. AEV-T.1.6.1 Forms and causes of removal from service or improper behavior of involute gears [Jula, 1989; Moldovean, 2001; Danape, 2012]

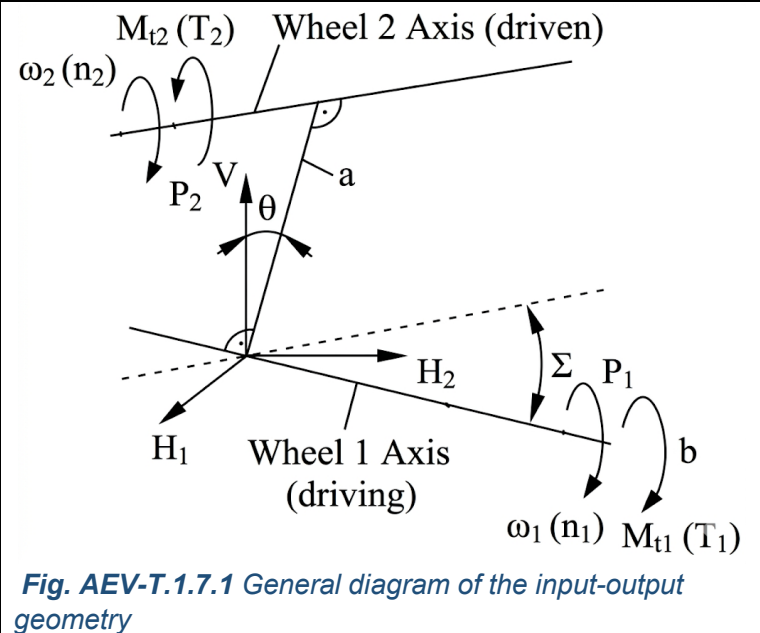
Failure / damage type	shapes	Appearance	CAUSE	Manifestation	Avoidance
Failure mode	Overload failure	For wheels made of steels with increased hardness	Overloads and shocks that occur under unusual operating conditions	Breakage at the base of the tooth, in straight-toothed wheels; breakage at the corner of the tooth in helical-tooth wheels (fig. AEV-T.1.6.1,a)	Limiting the transmitted torque by introducing safety couplings Increasing the precision of gear wheels and/or shaft rigidity
	Fatigue fracture	For wheels made of steel with a hardness greater than 45 HRC or cast iron	Exceeding the fatigue resistance of the material under time-varying bending stress	Initially, microcracks in the tensioned fillet area, which develop causing the teeth to break (fig. AEV-T.1.6.1,b)	Limitation by calculation of the maximum bending stresses at the tooth base to permissible fatigue values Providing large fillet radii Increasing the tooth section at the base by increasing the module and/or by positive profile displacements
	Plastic deformation	For wheels made of steels with low hardness	Overload and shock loads	Exceeding the yield strength of the material (fig. AEV-T.1.6.1,c)	Limitation of the transmitted torque, e.g. by introducing safety couplings
Abnormal behaviour (with irregularities, vibrations and noise)	Flank damage due to contact fatigue (pitting, pinching)	Especially for wheels made of steels with low surface hardness (below 55 HRC)	Material fatigue of the superficial layers of the active flanks determined by contact stresses varying over time	Initial microcracks on the surface of the active flanks that increase as a consequence of the action of the hydrostatic pressure of the oil generated during tooth contacts followed by the detachment of small particles of material resulting in small pits (pitting phenomenon) (fig. AEV-T.1.6.1,d; see fer. AEV-T.2) Progression of pitting over time and consequently abnormal operation with vibrations and noise	Limitation by calculation of the maximum contact stresses (fatigue) of the active flanks to permissible stresses Thermal and thermochemical treatments that lead to an increase in surface contact resistance Positive profile offsets for increasing profile radii Reducing the roughness of the active flanks Using additive lubricants.
	Flank damage due to scuffing	On heavily loaded gear wheels at high speeds and with improper lubrication	Increased slippage between flanks; high roughness of flanks	Scratches oriented along the flank (fig. AEV-T.1.6.1,e) generated by the formation and rupture of microwelds due to high local loads and high temperatures	Improving lubrication conditions (lubricants with additives); increasing manufacturing and assembly precision; reducing the roughness of the active flanks

	Flank damage due to abrasive wear	On open (unprotected) and improperly lubricated gear wheels	Increased slippage between flanks; high roughness of flanks	Removal of fine particles of material from the tooth flank as a result of abrasive particles existing in the contact area (fig. AEV-T.1.6.1,f)	Proper lubrication, seals and protection to the action of abrasive particles from the external environment
	Flank damage due to spalling	For wheels made of surface heat-treated or thermochemically treated steels (hardening, carburizing, nitriding)	Material fatigue of the superficial layers of the active flanks	The appearance of microcracks at the boundary between the hardened layer and the base layer (fig. AEV-T.1.6.1,g)	Appropriate thermal or thermochemical treatment technologies
	Flank damage due to plastic flow	For wheels made of steels with low hardness	Overloads, high slippage and low lubrication	The appearance of dents on the flanks of the driving wheel teeth and dents on the flanks of the driven wheel teeth as a result of high tangential stresses caused by frictional forces that change their direction in the gearing pole (fig. AEV-T.1.6.1, h)	Reducing loads and/or increasing flank hardness



AEV-T.1.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

Tab. AEV-T.1.7.1 Main input-output and construction parameters

 Fig. AEV-T.1.7.1 General diagram of the input-output geometry	Input-output parameters	
	$H_1 H_2 V$ – orthogonal spatial coordinate system with axes H_1 and H_2 in the horizontal plane and V in the vertical plane a – distance between the wheel axes θ – angle of the wheel center line with the vertical Σ – the angle between the wheel axes $\omega_{1,2} (n_{1,2})$ – angular velocity (speed) of the driving and driven wheels, respectively $M_{t1,2} (T_{1,2})$ – torque of the driving or driven wheel $P_{1,2}$ – power at the axis of the driving or driven wheel ($P_{1,2} = M_{t1,2} \omega_{1,2}$) $\eta = P_2 / P_1$ – gear efficiency $i = \pm \omega_1 / \omega_2 = \pm n_1 / n_2$ – <u>transmission ratio</u> ($-$) – external gear, ($+$) – internal gear)	
	Main construction parameters of the teeth	
	$z_{1,2}$ – numbers of teeth of the wheels ($i = \pm z_2 / z_1$) $u = \max(z_2, z_1) / \min(z_2, z_1)$ – <u>gear ratio</u> ($u = i$, for reduction gears ($n_1 \geq n_2$); $u = 1/ i$, for multiplier gears ($n_1 < n_2$); <u>pinion</u> – the wheel with the minimum number of teeth) $b_{1,2}$ – wheel widths	

AEV-T.2 MODELING OF INVOLUTE GEARS

AEV-T.2.1 MODELING OF STRAIGHT-TOOTH CYLINDRICAL GEARS

AEV-T.2.1.1 LOAD MODELING

Tab. AEV-T.2.1.1.1 Calculation relations for forces in spur gear

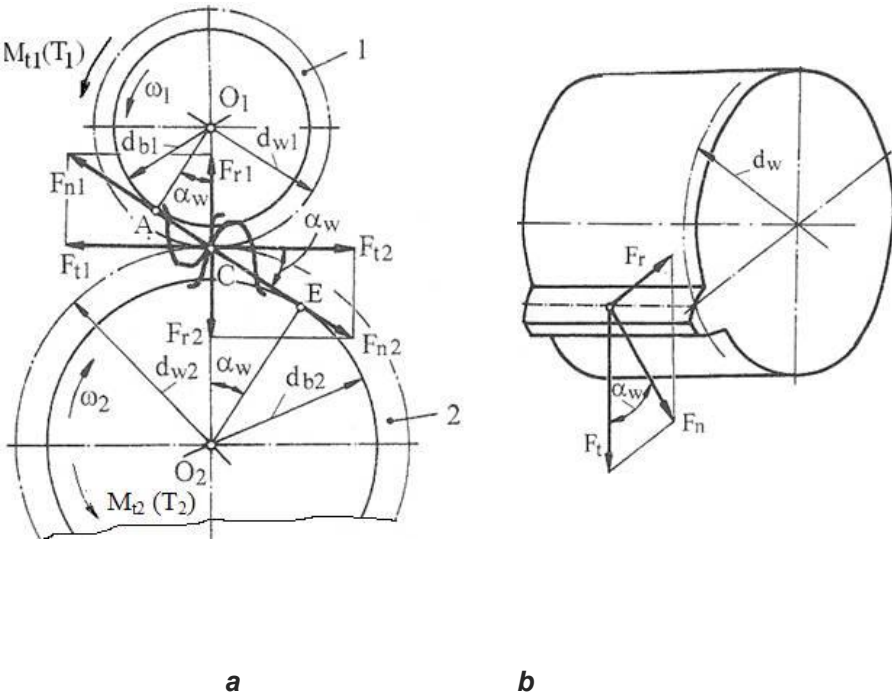
 a b	Simplifying assumptions: - normal forces are considered applied to the gear pole C at the middle of the wheel (fig. AEV-T.2.1.1.1,b), - friction is neglected, - the forces applied are considered static. Tab. AEV-T.2.1.1.1 Force calculation relationships	
Force <u>Tangential force,</u> $F_t = F_{t1} = F_{t2}$	The relationship $F_t = \frac{2 T_1}{d_{w1}} \quad (\text{AEV-T.2.1.1.1})$	Direction and meaning Direction tangent to the pitch circles; opposite direction to the speed (resistive force), for the driving wheel, and same direction as the speed (motor force), for the driven wheel
<u>Radial force,</u> $F_r = F_{r1} = F_{r2}$	The relationship $F_r = F_t \operatorname{tg} \alpha_w = \frac{2 T_1}{d_{w1}} \operatorname{tg} \alpha_w \quad (\text{AEV-T.2.1.1.2})$	Direction and meaning Radial direction; direction towards the center of the wheel
<u>Normal force,</u> $F_n = F_{n1} = F_{n2}$	The relationship $F_n = \frac{F_t}{\cos \alpha_w} = \frac{2 T_1}{d_{w1} \cos \alpha_w} \quad (F_n = \sqrt{F_r^2 + F_t^2}) \quad (\text{AEV-T.2.1.1.3})$	Direction and meaning Direction according to the common normal of the profiles in contact; opposite direction to the speed (resisting force), for the driving wheel, and the same direction as the speed (driving force), for the driven wheel

Fig. AEV-T.2.1.1.1 Force diagram [Moldovean, 2001]: **a** – in the frontal plane, **b** - spatial

Fig. AEV-T.2.1.1.1 Force diagram [Moldovean, 2001]: **a** – in the frontal plane, **b** - spatial

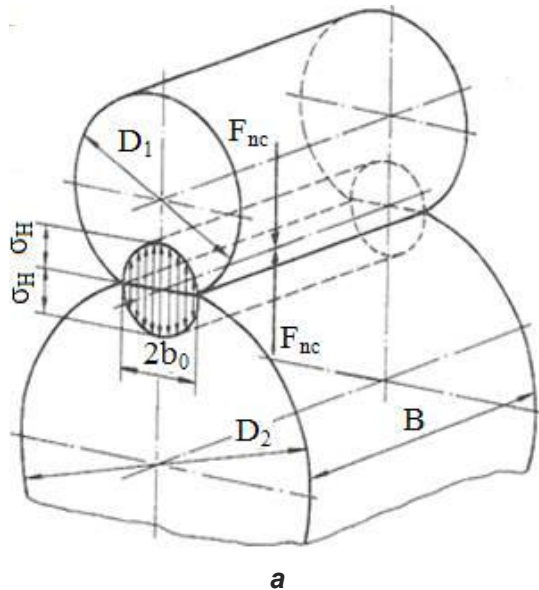
Note.

a. Meanings of the parameters in fig. AEV-T.2.1.1.1: $d_{w1,2}$ - diameters of the pitch circles; $d_{b1,2}$ - diameters of the base circles; α_w - actual gearing angle; $\omega_{1,2}$ - angular velocities; M_{t1} or T_1 - pinion torque; M_{t2} or T_2 - wheel torque.

b. For the unshifted gear ($\alpha_w = \alpha$; $d_{w1} = d_1$): $F_t = \frac{2 T_1}{d_1}$, $F_r = \frac{2 T_1}{d_{w1}} \tan \alpha$, $F_n = \frac{2 T_1}{d_{w1} \cos \alpha}$; α – pressure angle, d_1 - pitch diameter of the pinion.

AEV-T.2.1.2 CONTACT STRESS CALCULATION MODEL

Tab. AEV-T.2.1.2.1 Model and calculation relationship at contact [Jula, 1989; Moldovean, 2001; Rădulescu, 1985]

**The calculation model at reference contact [Hertz, 1881] (fig. AEV-T.2.1.2.1,a)**

In the contact area of two cylinders with diameters $D_{1,2}$ and lengths B , loaded with force F_{nc} , maximum normal contact stresses appear which, according to Hertz, are determined by the relationship,

$$\sigma_H = Z_E \sqrt{\frac{F_{nc}}{l_k} \frac{1}{\rho}}, \quad (\text{AEV-T.2.1.2.1})$$

where, $l_k = B$, represents the length of the contact line;

$$\frac{1}{\rho} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{1}{D_1/2} \pm \frac{1}{D_2/2},$$

reduced curvature ($\rho_{1,2}$ are the radii of curvature of the two cylinders, the sign (+) corresponds to the external contact (fig. AEC-T.1,a), and the sign (-) to the internal contact;

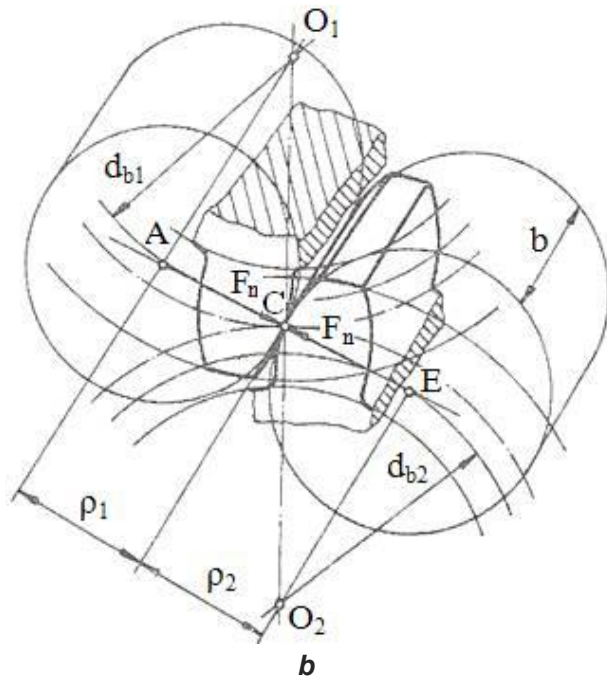
$$Z_E = \sqrt{\frac{1}{\pi \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)}}$$

the elasticity factor where $E_{1,2}$ and $v_{1,2}$ are the elastic moduli and, respectively, the transverse contraction coefficients (Poisson) of the materials of the two cylinders (ref. AEV-F.1).

Assumptions of Hertz's model:

- the cylinder materials are homogeneous, isotropic, elastic according to Hooke's law;
- the normal force is applied statically;
- contact stresses are distributed uniformly over the contact length;
- the width of the contact surface $2b_0$, as a result of the elastic deformation, is very small in relation to the dimensions of the cylinders;
- the cylinder surfaces are perfectly smooth;
- friction forces are not taken into account.

Differences between the model of two teeth in gearing (fig. AEV-T.1, b) and Hertz's model:



- the radii of curvature of the teeth are variable (the profiles are involute);
- the interaction force between the teeth does not act statically but variably depending on external and internal dynamic loads;
- contact stresses are not uniform along the contact line due to inaccuracies in execution and assembly and elastic deformations of the gear elements;
- there are frictional forces;
- the flanks are not perfect, they may have various roughnesses.

Customizing the parameters of Hertz's relation for the offset gear with straight teeth in contact at the gear pole:

Normal contact force,

$$F_{nc} = K_A K_V K_{H\beta} K_{H\alpha} F_n = K_A K_V K_{H\beta} K_{H\alpha} \frac{F_t}{\cos \alpha_w} = K_A K_V K_{H\beta} K_{H\alpha} \frac{2 T_1}{d_{w1} \cos \alpha_w}$$

(AEV-T.2.1.2.2)

K_A – operating mode factor, takes into account the effect of variable external loads (fer. [AEV-F.2](#)); K_V – internal dynamic factor, takes into account internal dynamic loads ([AEV-F.3](#)); $K_{H\beta}$ – factor of non-uniform load distribution along the contact line, specific to contact stress, takes into account execution errors in the tooth direction and elastic deformations of the gear subassembly elements ([AEV-F.4](#)); $K_{H\alpha}$ – factor of non-uniform load distribution in the frontal plane, specific to contact stress, takes into account pitch errors (fer. [AEV-F.5](#)).

Reduced curvature

Taking into account that,

$$\rho_1 = |AC| = d_{b1}/2 \operatorname{tg} \alpha_w \text{ and } \rho_2 = |EC| = d_2/2 \operatorname{tg} \alpha_w;$$

$$d_{b1,2} = d_{1,2} \cos \alpha \quad (d_{1,2} - \text{division diameters, } u = d_2/d_1), \text{ it results,}$$

$$\frac{1}{\rho} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{2}{\cos \alpha \operatorname{tg} \alpha_w} \frac{u \pm 1}{d_2} = \frac{2}{d_1 \cos \alpha \operatorname{tg} \alpha_w} \frac{u \pm 1}{u} \quad (\text{AEV-T.2.1.2.3})$$

Contact line length,

$$l_k = b/Z_\epsilon^2, \quad (\text{AEV-T.2.1.2.4})$$

where b is the tooth width (tooth length, wheel thickness); Z_ϵ – the contact ratio factor specific to the contact stress (fer. [AEV-F.6](#))

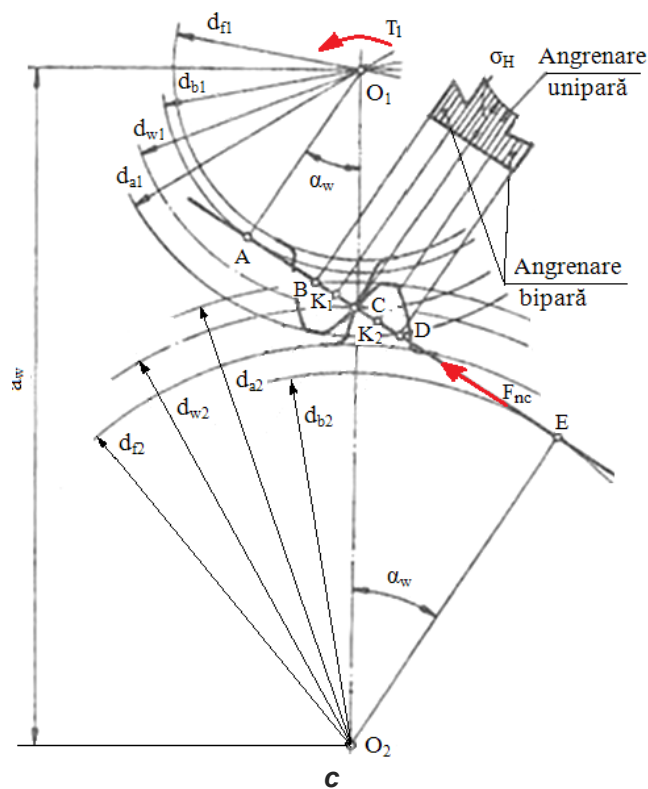


Fig. AEV-T.2.1.2.1 Contact calculation model of the spur gear: **a** – reference calculation model (Hertz), **b** – spatial calculation model **c** – frontal plane calculation model

Tab. AEV-T.2.1.2.2 Contact calculation relations (extended forms of the basic calculation relation (AEV-T.2.1.2.1))

Maximum contact stress	Verification	sizing
$\sigma_H = Z_E Z_\epsilon Z_H \sqrt{\frac{2T_1}{b d_1^2} K_A K_V K_H \beta K_{H\alpha} \frac{u+1}{u}} \quad (\text{AEV-T.2.1.2.5})$	$\sigma_H \leq \sigma_{HP1,2}, \quad (\text{AEV-T.2.1.2.8})$	$d_1 = \sqrt[3]{\frac{2T_1 K_A K_V K_H \beta K_{H\alpha}}{\psi_d \sigma_{HP}^2} (Z_E Z_\epsilon Z_H)^2 \frac{u+1}{u}} \quad (\text{AEV-T.2.1.2.9})$
$\sigma_H = \frac{Z_E Z_\epsilon Z_H}{m z_1} \sqrt{\frac{2T_1}{b} K_A K_V K_H \beta K_{H\alpha} \frac{u+1}{u}} \quad (\text{AEV-T.2.1.2.6})$	$\sigma_{HP1,2} = \frac{\sigma_{H \lim} Z_{N1,2}}{S_{H \min}} Z_L Z_R Z_V Z_W Z$ $\sigma_{HP1,2}$ – permissible contact stresses for the pinion and, respectively, the wheel;	$m = \sqrt[3]{\frac{2T_1 K_A K_V K_H \beta K_{H\alpha}}{\psi_m z_1^2 \sigma_{HP}^2} (Z_E Z_\epsilon Z_H)^2 \frac{u+1}{u}} \quad (\text{AEV-T.2.1.2.10})$

$\sigma_H = \frac{Z_E Z_\epsilon Z_H}{a_w} \sqrt{\frac{T_1}{2b} K_A K_v K_{H\beta} K_{H\alpha} \frac{(u \pm 1)}{u}} \quad (\text{AEV-T.2.1.2.7})$	σ_{Hlim} – <u>limit contact stress</u> (fer. AEV-M.1); S_{Hmin} – <u>minimum safety factor for contact</u> (fer. AEV-F.14); Z_x – <u>dimensional factor</u> (fer. AEV-F.8); Z_w – <u>hardness factor</u> (fer. AEV-F.9); Z_v – <u>speed factor</u> (fer. AEV-F.10); Z_R – <u>flank roughness factor</u> (fer. AEV-F.11); Z_L – <u>lubricant factor</u> (fer. AEV-F.12); $Z_{N1,2}$ – <u>factors of the number of load cycles of the pinion and the wheel</u> (fer. AEV-F.13); $\sigma_{HP} = \min(\sigma_{HP1}, \sigma_{HP2})$ – <u>permissible contact stress of the gear</u>	$a_w = (u \pm 1)^3 \sqrt{\frac{T_1 K_A K_v K_{H\beta} K_{H\alpha}}{2 \psi_a u \sigma_{HP}^2} (Z_E Z_\epsilon Z_H)} \quad (\text{AEV-T.2.1.2.11})$
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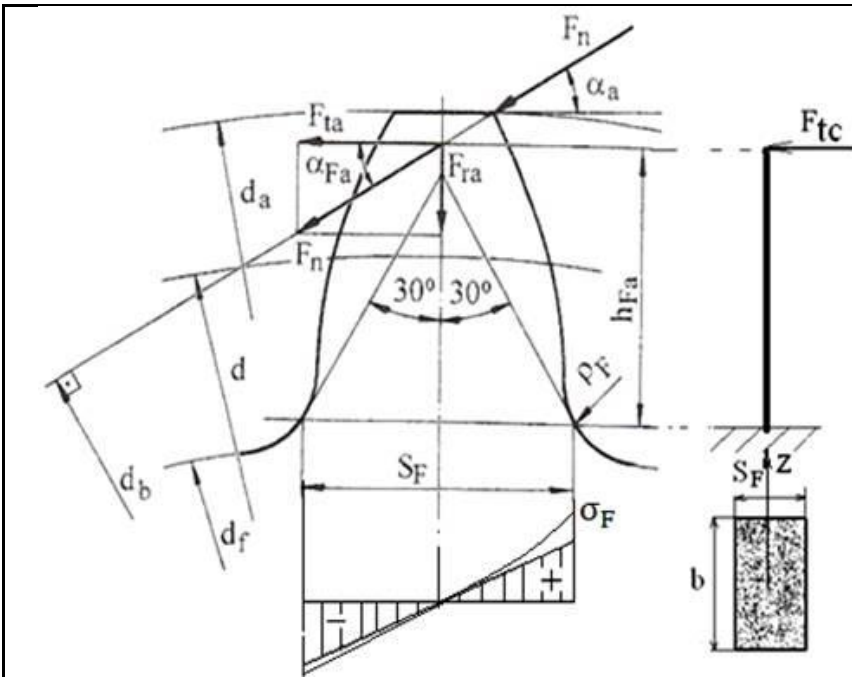
Note.

a. In the relations in this table $Z_H = \sqrt{\frac{2}{\cos^2 \alpha \tan \alpha_w}}$ is the contact area factor (fer. [AEV-F.7](#)); m – tooth module (fer. [AEV-P.1.4](#)); z_1 – number of teeth of the pinion (fer. [AEV-P.1.1](#)); $\psi_a = b/a_w$, $\psi_m = b/m$, $\psi_d = b/d_1$ – width factors related to the distance between the axes, the module and, respectively, the diameter of the pinion (fer. [AEV-P.1.3](#)); to simplify the calculation relations, $\cos \alpha / \cos \alpha_w \approx 1$

b. For the unshifted gear ($d_{w1} = d_1$; $\alpha_w = \alpha$): $d_1 = \frac{2a}{u \pm 1}$; $d_1 = m z_1$ (see fer [AEV-T.8.1](#)); $Z_H = \sqrt{\frac{2}{\cos \alpha \sin \alpha}}$ (fer. [AEV-F.7](#)).

AEV-T.2.1.3 BENDING STRESS CALCULATION MODEL

Tab. AEV-T.2.1.3. Model and basic calculation relation for bending [Jula, 1989; Rădulescu, 1985; Moldovean, 2001]



Simplifying assumptions:

- the tooth is considered a straight bar subjected to bending stress, taking into account the stress concentrator at the base of the tooth (fig. AEV-T.2.1.3.1),
- the normal force is considered concentrated at the tip of the tooth,
- the compressive stress generated by the radial force is neglected,
- the calculated thickness of the tooth is considered to be determined by two tangents to the fillet profiles that make 30 ° with the tooth axis.

Theoretical design bending force (fig. AEV-T.2.1.3.1),

$$F_{tc} = F_{ta} K_A K_v K_{F\beta} K_{F\alpha} = F_n \cos \alpha_a K_A K_v K_{F\beta} K_{F\alpha} = \frac{2 T_1 \cos \alpha_a}{d_{w1} \cos \alpha}$$

$$K_A K_v K_{F\beta} K_{F\alpha}, \text{ (AEV-T.2.1.3.1)}$$

where K_A – operating mode factor, takes into account the effect of external loads (fer. AEV-F.2); K_v – internal dynamic factor, takes into account internal dynamic loads (fer. AEV-F.3); $K_{F\beta}$ – factor of non-uniform load distribution along the contact line, specific to bending stress, takes into account execution errors from the tooth direction and elastic deformations of the gear subassembly elements (fer. AEV-F.4); $K_{F\alpha}$ – factor of non-uniform load distribution in the frontal plane, specific to bending stress, takes into account pitch errors (fer. AEV-F.5).

The reference calculation relationship for the maximum bending stress,

$$\sigma_F = \frac{M_i}{W_z} = \frac{F_{tc} h_{Fa}}{b S_F^2 / 6} Y_\epsilon Y_{Sa} = \frac{2 T_1}{d_1 b m} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_{Sa} Y_{Fa}, \text{ (AEV-T.2.1.3.2)}$$

$$6 \frac{h_{Fa}}{m} \cos \alpha_a$$

with $Y_{Fa} = \left(\frac{S_F}{m} \right)^2 \cos \alpha$, the form factor (fer. AEV-F.7); Y_ϵ – the contact ratio factor for bending stress (fer. AEV-F.6); Y_{Sa} – bending stress concentration factor at the tooth base (fer. AEV-F.15)

Tab. AEV-T.2.1.3.2 Bending calculation relations (extended forms of the basic calculation relation (AEV-T.2.1.3.2))

Maximum bending stresses (1 – pinion, 2- wheel)	Verification	sizing
$\sigma_{F1,2} = \frac{2 T_1 z_1}{b_{1,2} d_1^2} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_{Sa1,2} Y_{Fa1,2} \text{ (AEV-T.2.1.3.3)}$	$\sigma_{F1} \leq \sigma_{FP1}, \sigma_{F2} \leq \sigma_{FP2} \text{ (AEV-T.2.1.3.7)}$	$d_1 = \sqrt[3]{\frac{2 T_1 z_1}{\Psi_d} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon \left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)} \text{ (AEV-T.2.1.3.8)}$

$\sigma_{F1,2} = \frac{2 T_1}{b_{1,2} m^2 z_1} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_{Sa1,2} Y_F \quad (\text{AEV-T.2.1.3.4})$	$\sigma_{FP1,2} = \frac{\sigma_{F \lim 1,2} Y_{N1,2} Y_{ST}}{S_{F \min}} Y_{\delta 1,2} Y_{R1,2}$	$m = \sqrt[3]{\frac{2 T_1}{\Psi_m z_1} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon \left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_n} \quad (\text{AEV-T.2.1.3.9})$
$\sigma_{F1,2} = \frac{T_1 z_1 (u \pm 1)^2}{2 b_{1,2} a_w^2} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_{Sa1,2} \quad (\text{AEV-T.2.1.3.5})$	$\sigma_{FP1,2}$ – allowable bending stresses for pinion and wheel; $\sigma_{F \lim 1,2}$ – limit bending stress for pinion and wheel (fer. AEV-M.1); $S_{F \min}$ – minimum safety factor for bending (fer. AEV-F.1); $Y_{N1,2}$ – factor of the number of load cycles of the pinion and the wheel (fer. AEV-F.13), Y_{ST} – reference bending stress factor ($Y_{ST}=2$); $Y_{\delta 1,2}$ – stress concentrator factors of the pinion and wheel (fer. AEV-F.15); $Y_{R1,2}$ – roughness factors of the pinion and wheel fillets (fer. AEV-F.11); Y_x – gear size factor for bending (fer. AEV-F.8) $\left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max} = \max \left(\frac{Y_{Sa1} Y_{Fa1}}{\sigma_{FP1}}, \frac{Y_{Sa2}}{\sigma_{FP2}} \right)$	$a_w = \sqrt[3]{\frac{T_1 z_1 (u \pm 1)^2}{2 \Psi_a} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon \left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max}} \quad (\text{AEV-T.2.1.3.10})$

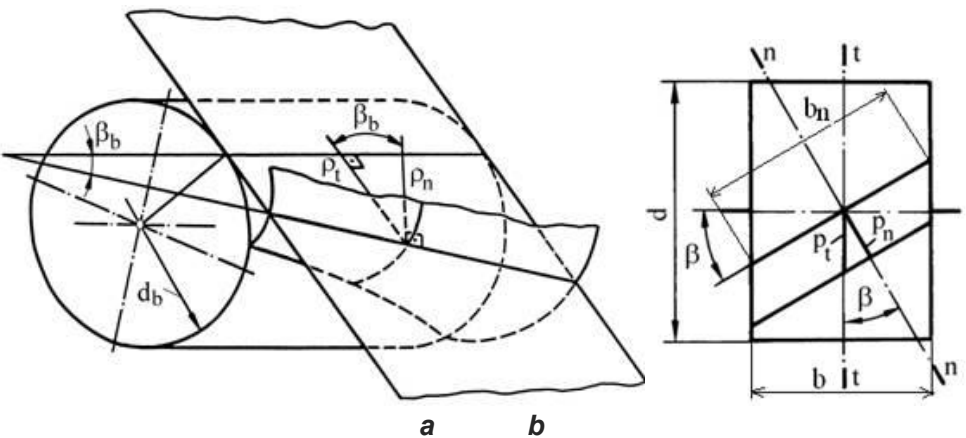
Note.

- a.** For the non-displaced gear ($d_{w1} = d_1$; $\alpha_w = \alpha$; $a_w = a$): $d_1 = \frac{2 a}{u \pm 1}$; $d_1 = m z_1$ (fer. [AEV-F.11](#)); m – tooth module (fer. [AEV-P.1.4](#)), z_1 – number of teeth of the pinion (fer. [AEV-P.1.1](#));
- b.** In the relationships in this table: $\Psi_a = b/a$, $\Psi_m = b/m$, $\Psi_d = b/d_1$ – width factors related to the distance between the axes, the module and, respectively, the diameter of the pinion (fer. [AEV-P.1.3](#))

AEV-T.2.2 MODELING OF CYLINDRICAL GEARS WITH INCLINED TEETH

AEV-T.2.2.1 GEOMETRIC AND FUNCTIONAL PARTICULARITIES OF CYLINDRICAL GEAR WITH INCLINED TEETH

Tab. AEV-T.2.1.1.1 Geometric, functional and constructive features [Jula, 1989; Rădulescu, 1985; Moldovean, 2001]

 <i>Fig. AEV-T.2.2.1.1 Main geometric characteristics of the inclined involute profile: a - spatial view; b – axial view</i>	Geometric features - The tooth flank is generated by moving a straight line inclined at an angle β_b with respect to the generator of the base cylinder (fig. AEV-T.2.2.1.1,a). The dependence between the radii of curvature of the involute (profile) in the normal plane n-n, ρ_n, and that in the frontal plane t-t, ρ_t, is $\rho_n = \rho_t \cos \beta_b$. (AEV-T.2.2.1.1) - The dependence between the tooth pitch in the frontal plane p_t and the one in the normal plane p_n (fig. AEV-T.2.2.1.1,b) is $p_n = p_t \cos \beta$; taking into account that $p = m_t \pi$, the dependence between the frontal module m_t and m_n results, $m_n = m_t \cos \beta$, (AEV-T.2.2.1.2) with β the angle of inclination of the teeth; Note: in design practice, usually, geometric parameters in the frontal plane do not contain the index t ($m_t = m$, $p_t = p$, $\rho_t = \rho$ etc.)
	Constructive and functional features - The length of the tooth is greater than the width of the wheel (bending resistance is greater than in the case of a straight tooth). - The inclined teeth progressively enter and exit the gearing, respectively, which leads to reduced noise and, therefore, use for higher speeds. - There are several pairs of teeth in the gear; <u>the total contact ratio</u>, $\varepsilon_v = \varepsilon_\alpha + \varepsilon_\beta$, ε_α – <u>the contact ratio in the frontal plane</u>, ε_β – <u>the additional contact ratio</u>, due to the inclination of the teeth.

AEV-T.2.2.2 LOAD MODELING [Rădulescu, 1985]

	Simplifying assumptions: - normal forces are considered applied to the gear pole C at the middle of the wheel width (fig. AEV-T.8.2.2.1) - friction is neglected, - forces are considered to be applied statically		
	Tab. AEV-T.2.2.2.1 Force calculation relationships		
	parameter	The relationship	Direction and sense (fig. AEV-T.2.2.2.2)

	Tangential force, $F_t = F_{t1} = F_{t2}$	$F_t = \frac{2 T_1}{d_{w1}} \text{ (AEV-T.2.2.2.1)}$	Direction tangent to the pitch circles; opposite direction to the speed (resistive force), for the driving wheel, and same direction as the speed (motor force), for the driven wheel
	Radial force, $F_r = F_{r1} = F_{r2}$	$F_r = F_t \operatorname{tg} \alpha_{wt} = \frac{2 T_1}{d_{w1}} \operatorname{tg} \alpha_{wt} \text{ (AEV-T.2.2.2.2)}$	Radial direction; direction towards the center of the wheel; $\tan \alpha_{wt} = \tan \alpha_{wn} / \cos \beta_w$
	Axial force, $F_a = F_{a1} = F_{a2}$	$F_a = F_t \operatorname{tg} \beta_w = \frac{2 T_1}{d_{w1}} \operatorname{tg} \beta_w \text{ (AEV-T.2.2.2.3)}$	Axial direction; the direction determined by the direction of inclination of the tooth and the direction of rotation of the wheel
	Normal force, $F_n = F_{n1} = F_{n2}$	$F_n = \frac{F_t}{\operatorname{tg} \beta_w \cos \alpha_{wn}} = \frac{2 T_1}{d_{w1} \operatorname{tg} \beta_w \cos \alpha_{wn}} \text{ (AEV-T.2.2.2.4)}$ $(F_n = \sqrt{F_r^2 + F_t^2 + F_a^2})$	Direction according to the common normal of the profiles in contact; opposite direction to the speed (resisting force), for the driving wheel, and the same direction as the speed (driving force), for the driven wheel
	Note 1 For the unshifted gear ($\alpha_{wn} = \alpha_n = \alpha$; $\beta_w = \beta$; $d_{w1} = d_1$):	$F_t = \frac{2 T_1}{d_1},$	$F_r = \frac{2 T_1}{d_1 \cos \beta} \operatorname{tg} \alpha,$
	$F_a = \frac{2 T_1}{d_1} \operatorname{tg} \beta,$	$F_n = d_1 \operatorname{tg} \beta \cos \alpha,$	$\alpha = \alpha_n$ – reference pressure angle, d_1 – pinion pitch diameter; β – tooth inclination angle

Fig. AEV-T.2.2.2.1 Force diagram: a – in the axial plane for the pinion, b – in the frontal plane for the gear; c – in the normal plane for the gear; d – in the axial plane for the wheel; e – spatial view

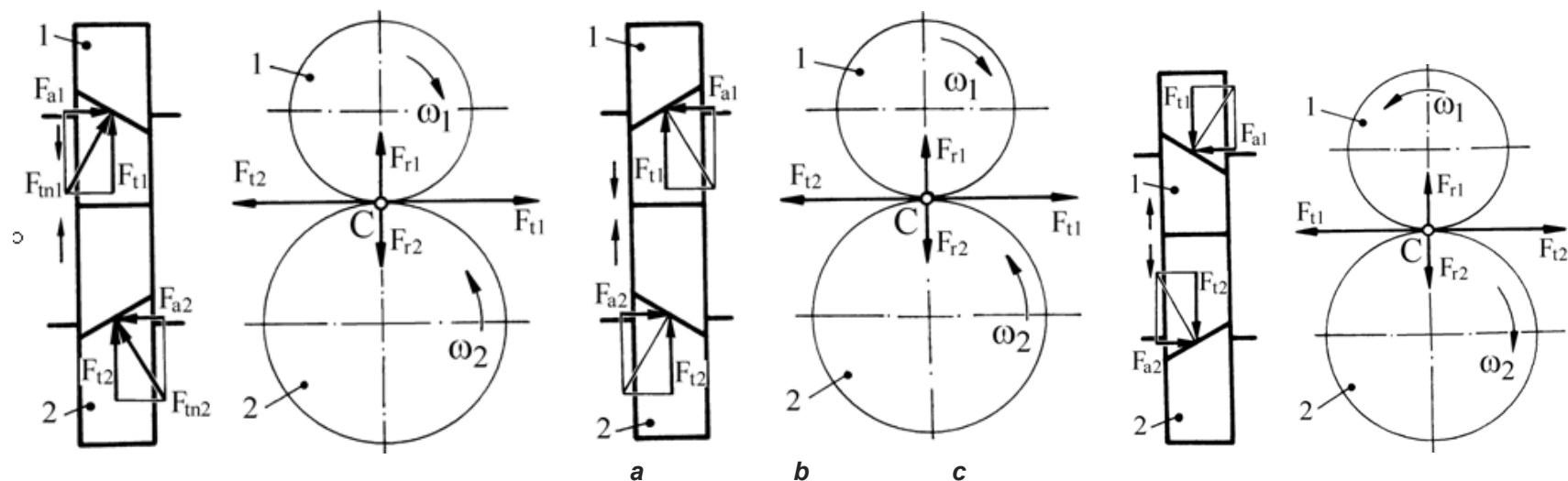
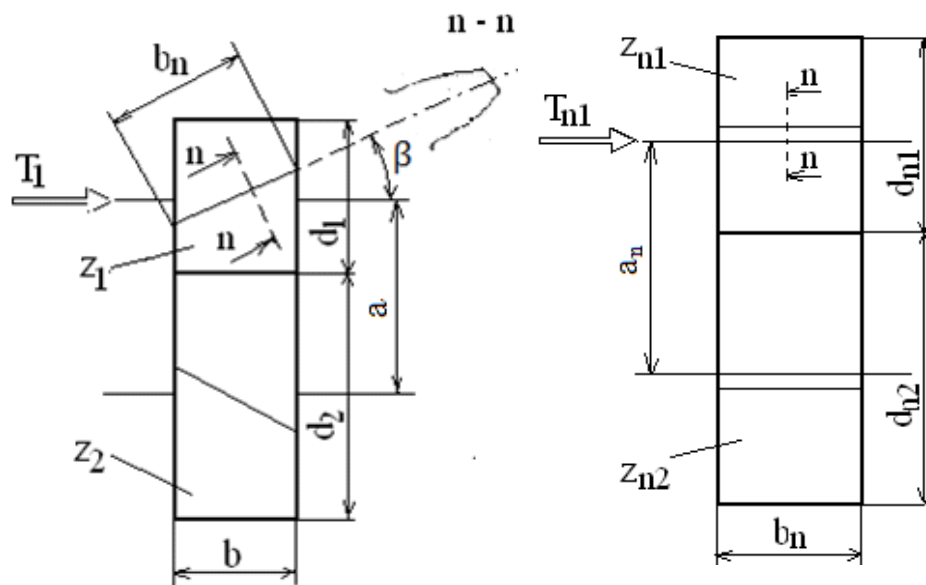


Fig. AEV-T.2.2.2.2 Directions and senses of forces in cylindrical gears with helical teeth: a – clockwise movement and inclination of the teeth to the right; b – clockwise movement and inclination of the teeth to the left; c – counterclockwise movement and inclination of the teeth to the right [Moldovean, 2001]

Note 2 Meanings of the geometric parameters in Fig. AEV-T.2.2.2.1: $d_{w1,2}$ - diameters of the pitch circles; β_w – angle of inclination of the teeth on the pitch circle; α_{wt} - angle of engagement in the frontal plane; α_{wn} - angle of engagement in the normal plane; $\omega_{1,2}$ - angular velocities; M_{t1} or T_1 - torque of the pinion; M_{t2} - torque of the wheel; F_{tn} – normal force in the tangential plane ($F_{tn} = F_{tn1} = F_{tn2}$)

AEV-T.2.2.3 CONTACT STRESS CALCULATION MODEL

Tab. AEV-T.2.2.3.1 Definition and parameters of the equivalent gear [Jula, 1989; Rădulescu, 1985; Moldovean, 2001]



Definition of equivalent gear (fictitious, virtual):

Since in the plane normal to the tooth direction the dimensions are minimal and, in addition, the interaction force F_n acts in this plane, the calculation of the cylindrical gear with helical teeth is performed by means of an equivalent cylindrical gear with equivalent wheels (with straight teeth, fig. AEV-T.2.2.3.1,a,b), which has the same geometric and loading parameters as the real gear (with helical teeth) in the normal plane.

The equivalent wheel is the spur gear wheel that has teeth with the same shape and dimensions as the teeth of the real wheel in the normal plane (hence the index n); for mathematical modeling, the following equalities will be considered: the radius of curvature of the tooth profile of the equivalent wheel is equal to the radius of curvature of the tooth profile of the real wheel in the normal plane; the module of the equivalent wheel is equal to the normal module of the real wheel; the coefficient of profile displacement of the equivalent wheel is equal to the coefficient of profile displacement of the real wheel in the normal plane.

Parameters of the equivalent cylindrical gear (for the unshifted helical gear):

Diameters of the division circles

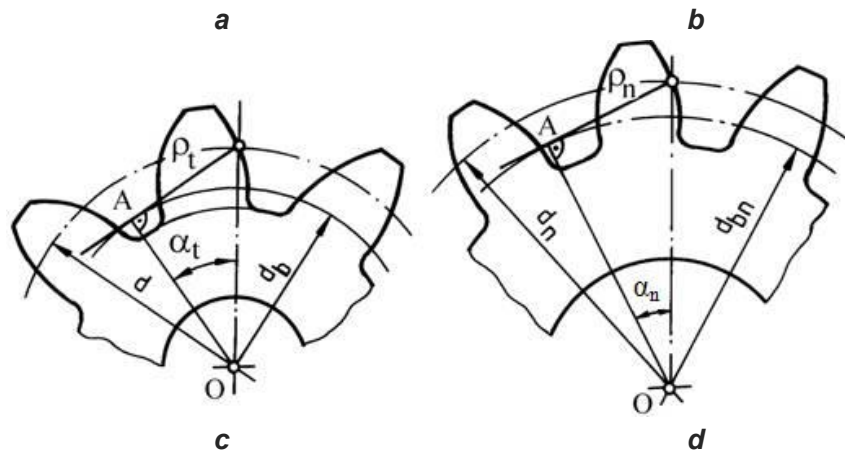


Fig. AEV-T.2.2.3.1 Equivalence model of a helical gear with a spur gear: a - axial view of the helical gear (real), b - axial view of the equivalent spur gear (virtual), c - front view of the helical pinion/wheel (real); d - front view of the equivalent spur pinion/wheel (virtual)

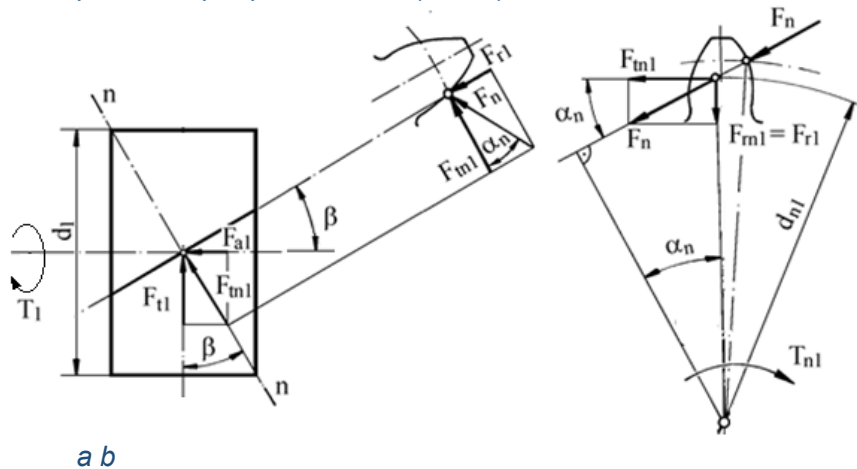


Fig. AEV-T.2.2.3.2 Torque equivalence model: a - axial view and normal section of the pinion tooth (real), b - front view of the equivalent pinion

From Fig. AEV-T.2.2.3.1, c and d it results $d = 2 \rho_t \sin \alpha_t$ and, respectively, $d_n = 2 \rho_n \sin \alpha_n$ (the relations are general, if necessary, they can be customized for the pinion, 1 and the wheel 2); taking into account the relation (AEV-T.2.2.1.1), it results

$$d_n = d \frac{\sin \alpha_t}{\sin \alpha \cos \beta_b} = d \frac{1}{\cos^2 \beta_b} \quad (\text{AEV-T.2.2.3.1})$$

The numbers of teeth on the wheels

Substituting in the relation (AEV-T.2.2.3.1), $d_n = m_n z_n$ and $d = m z$ and, in addition, taking into account the relation (AEV-T.2.2.1.2), it results

$$z_n = z \frac{1}{\cos \beta \cos^2 \beta_b} \quad (\text{AEV-T.2.2.3.2})$$

Distance between axes

Taking into account the relation (AEV-T.2.2.3.1),

$$a_n = \frac{d_{n1} + d_{n2}}{2} = \frac{d_1 + d_2}{2} \frac{1}{\cos^2 \beta_b} = a \frac{1}{\cos^2 \beta_b} \quad (\text{AEV-T.2.2.3.3})$$

Gear ratio

Considering rel. (AEV-T.2.2.3.3),

$$u_n = \frac{z_{n2}}{z_{n1}} = \frac{z_2 / (\cos \beta \cos^2 \beta_b)}{z_1 / (\cos \beta \cos^2 \beta_b)} = \frac{z_2}{z_1} = u \quad (\text{AEV-T.2.2.3.4})$$

Pinion torque

Taking into account Fig. AEV-T.2.2.3.2 and the relation (AEV-T.2.2.3.3), it results

$$T_{n1} = F_{tn1} \frac{d_{n1}}{2} = \frac{F_{t1}}{\cos \beta} \frac{d_1}{2 \cos^2 \beta_b} = F_{t1} \frac{d_1}{2} \frac{1}{\cos \beta \cos^2 \beta_b} = T_1 \frac{1}{\cos \beta \cos^2 \beta_b} \quad (\text{AEV-T.2.2.3.5})$$

Tooth width (fig. AEV-T.2.2.3.1, a, b)

$$b_n = \frac{b}{\cos \beta} \quad (\text{AEV-T.2.2.3.6})$$

The contact calculation of the helical gear is done through the equivalent spur gear.

The calculation relations (tab. AEV-T.2.2.3.2) for contact of the cylindrical gear with helical teeth obtained by replacing the parameters of the equivalent gear (tab. AEV-T.2.2.3.1) in the calculation relations for contact of the gear with straight teeth in chap. AEV-T.2.1, tab. AEV-T.2.1.2.2 are synthesized in tab. AEV-T.2.2.3.2.

Tab. AEV-T.2.2.3.2 Calculation relations for contact of cylindrical gear with helical teeth

Contact stress	Verification	sizing
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$\sigma_H = Z_E Z_\epsilon Z_H Z_\beta \sqrt{\frac{2T_1}{b d_1^2} K_A K_v K_{H\beta} K_{H\alpha} \frac{u \pm 1}{u}} \quad (\text{AEV-T.2.2.3.7})$	$\sigma_H \leq \sigma_{HP1,2}, (\text{AEV-T.2.2.3.10})$ $\sigma_{HP1,2} = \frac{\sigma_{H \lim} Z_{N1,2}}{S_{H \min}} Z_L Z_R Z_v Z_w$ $\sigma_{HP1,2}$ – permissible contact stresses for pinion and wheel, $\sigma_{H \lim}$ – limit contact stress (fer. AEV - M.1), $S_{H \min}$ – minimum contact safety factor (fer. AEV-F.1), $Z_{N1,2}$ – factors of the number of load cycles of the pinion and the wheel (fer. AEV-F.13), Z_R – flank roughness factor (see fer. AEV-F.11), Z_L – lubricant factor (fer. AEV-F.12), Z_v – speed factor (fer. AEV-F.10), Z_w – hardness factor (fer. AEV-F.9), Z_x – size factor (fer. AEV-F.8) $Z_\beta = \sqrt{\cos \beta}$ – tooth inclination factor for contact stress	$d_1 = \sqrt[3]{\frac{2T_1 K_A K_v K_{H\beta} K_{H\alpha}}{\psi_d \sigma_{HP}^2} (Z_E Z_\epsilon Z_H Z_\beta)^2 \frac{u \pm 1}{u}} \quad (\text{AEV-T.2.2.3.11})$ $m = \sqrt[3]{\frac{2T_1 K_A K_v K_{H\beta} K_{H\alpha}}{\psi_m} \left(\frac{Z_E Z_\epsilon Z_H Z_\beta}{\sigma_{HP} z_1} \right)^2 \frac{u \pm 1}{u}} \quad (\text{AEV-T.2.2.3.12})$ $a_w = (u \pm 1) \sqrt[3]{\frac{T_1 K_A K_v K_{H\beta} K_{H\alpha}}{2 \psi_a u \sigma_{HP}^2} (Z_E Z_\epsilon Z_H Z_\beta)^2} \quad (\text{AEV-T.2.2.3.13})$
$\sigma_H = \frac{Z_E Z_\epsilon Z_H Z_\beta}{a_w} \sqrt{\frac{T_1}{2b} K_A K_v K_{H\beta} K_{H\alpha} \frac{(u \pm 1)}{u}} \quad (\text{AEV-T.2.2.3.9})$		

Note.

- a.** For the non-displaced gear ($d_{w1} = d_1$; $\alpha_w = \alpha$): $d_1 = \frac{2a}{u \pm 1}$; $d_1 = m z_1$ (fer. AEV-T.1.2); m – front module of the toothing (fer. AEV-P.1.4), z_1 -number of teeth of the pinion (fer. AEV-P.1.1); $Z_H = \sqrt{\frac{2 \cos \beta_b}{\cos^2 \alpha \tan \alpha_w}}$, contact area factor (fer. AEV-F.7); to simplify the calculation relations, $\cos \alpha / \cos \alpha_w \approx 1$
- b.** In the relationships in this table: „- width factors related to the distance between the axles, the module and, respectively, the diameter of the pinion (fer. AEV-P.1.3) $\psi_a = b/a_w$ $\psi_m = b/m$ $\psi_d = b/d_1$
- c.** The calculation relations for the helical gear are almost the same as those for the spur gear, with the difference that, in addition, the factor Z_β appears; differences also appear in the calculation relations for the correction factors (Z_ϵ , Z_H)

AEV-T.2.2.4 BENDING STRESS CALCULATION MODEL

The bending calculation is also done using the equivalent spur gear. The bending calculation relations (tab. AEV-T.2.2.4.1) are obtained by replacing the parameters of the equivalent gear (tab. AEV-2.2.3.1) in the bending calculation relations of the spur gear in chap. AEV.T.2.1, tab. AEV-T.2.1.3.2.

Tab. AEV-T.2.2.4.1 Calculation relations for bending stresses

Maximum bending stresses (1 – pinion, 2- wheel)	Verification	sizing
$\sigma_{F1,2} = \frac{2 T_1 z_1}{b_{1,2} d_1^2 \cos\beta} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta Y_{Sa} \quad (\text{AEV-T.2.2.4.1})$	$\sigma_{F1} \leq \sigma_{FP1}, \sigma_{F2} \leq \sigma_{FP2}, (\text{AEV-T.2.2.4.4})$	$d_1 = \sqrt[3]{\frac{2 T_1 z_1}{\psi_d \cos\beta} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta \left(\frac{Y_{Sa} Y_F}{\sigma_{FP}} \right)} \quad (\text{AEV-T.2.2.4.5})$
$\sigma_{F1,2} = \frac{2 T_1}{b_{1,2} m^2 z_1 \cos\beta} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta Y_{Sa} \quad (\text{AEV-T.2.2.4.2})$	$\sigma_{FP1,2} = \frac{\sigma_F \lim_{1,2} Y_{N1,2} Y_{ST}}{S_F \min} Y_{\delta 1,2}$, $\sigma_{FP1,2}$ – allowable bending stresses for pinion and wheel; $\sigma_{Flim1,2}$ – limit bending stress for pinion and wheel (fer. AEV-M.1); S_{Fmin} – minimum safety factor for bending (fer. AEV-F.1); $Y_{N1,2}$ – factor of the number of load cycles of the pinion and the wheel (fer. AEV-F.13), Y_{ST} – reference bending stress factor ($Y_{ST}=2$); $Y_{\delta 1,2}$ – stress concentrator factors of the pinion and wheel (fer. AEV-F.15); $Y_{R1,2}$ – roughness factors of the pinion and wheel fillets (fer. AEV-F.11), Y_x – size factor for bending (fer. AEV-F.8); Y_β – tooth inclination factor for bending stress (fer. AEV-F.9);	$m = \sqrt[3]{\frac{2 T_1}{\psi_m z_1 \cos\beta} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta \left(\frac{Y_{Sa} Y}{\sigma_{Fl}} \right)} \quad (\text{AEV-T.2.2.4.6})$
$\sigma_{F1,2} = \frac{T_1 z_1 (u \pm 1)^2}{2 b_{1,2} a_w^2 \cos\beta} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta Y_{Sa1} \quad (\text{AEV-T.2.2.4.3})$		$a_w = \sqrt[3]{\frac{T_1 z_1 (u \pm 1)^2}{2 \psi_a \cos\beta} K_A K_v K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta \left(\frac{Y_s}{\sigma_c} \right)} \quad (\text{AEV-T.2.2.4.7})$

Note.

a. For the non-displaced gear ($d_{w1} = d_1$; $\alpha_w = \alpha$): $d_1 = \frac{2 a_w}{u \pm 1}$; $d_1 = m z_1$ (fer. [AEV-T.1.2](#)); m – front module of the toothings (fer. [AEV-P.1.4](#)), z_1 – number of teeth of the pinion (fer. [AEV-P.1.1](#)); $\left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max} = \max \left(\frac{Y_{Sa1} Y_{Fa1}}{\sigma_{FP1}}, \frac{Y_{Sa2} Y_{Fa2}}{\sigma_{FP2}} \right)$; to simplify the calculation relations, $\cos\alpha / \cos\alpha_w \approx 1$ is considered

b. In the relationships in this table: ψ_a – width factors related to the distance between the axles, the module and, respectively, the diameter of the pinion (fer. [AEV-P.1.4](#)) $\psi_a = b/a_w$ $\psi_m = b/m$ $\psi_d = b/d_1$

c. The calculation relations for the helical gear are almost the same as those for the spur gear, with the difference that, in addition, the Y_β factor appears, and $\cos\beta$ appears in the denominator of the fractions.

AEV-T.2.3 MODELING OF BEVEL GEARS (STRAIGHT, CURVED (INCLINED) AND HYPOID)

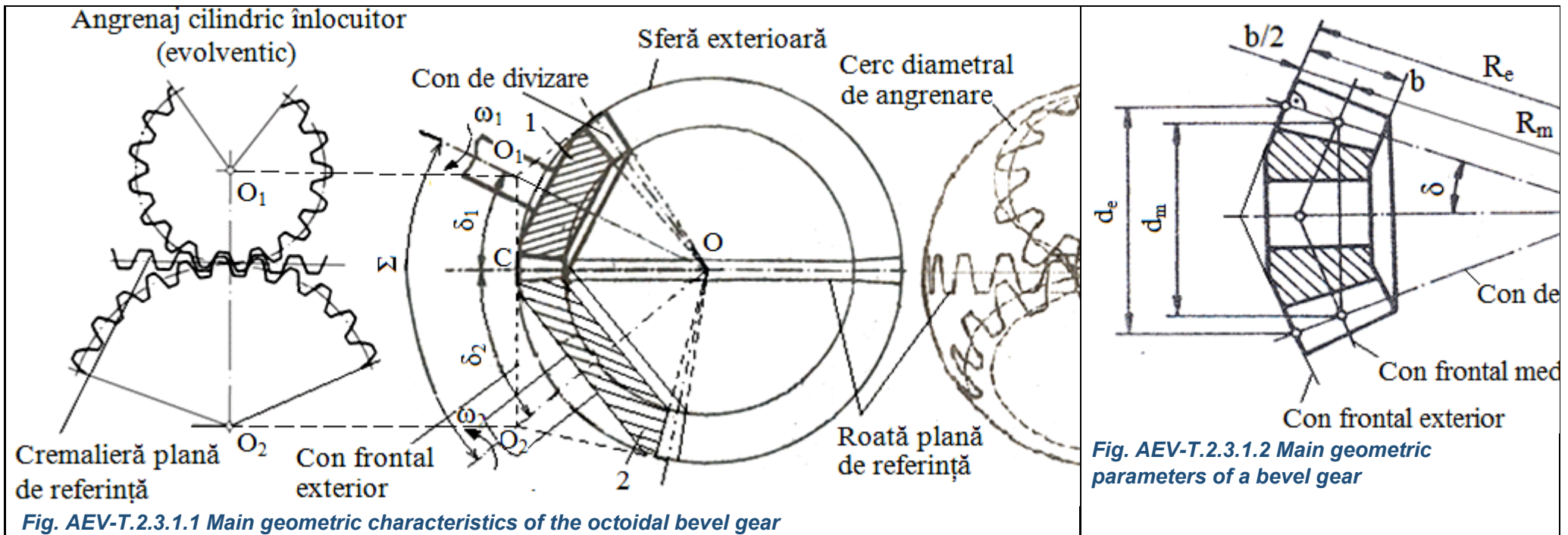
AEV-T.2.3.1 GEOMETRIC AND FUNCTIONAL PARTICULARITIES OF BEVEL GEARS [Rădulescu, 1985; Moldovean, 2001]

General aspects	Main kinematic and geometric parameters
- From the point of view of the spatial position of the general gear axes (tab. AEV-T.1.7.1), the <u>bevel gear</u> has the distance between the axes $a = 0$ and the angle between them $\Sigma \neq 0$. - The bevel gear is a <u>spherical gear</u> with the axes of motion circular conical surfaces with semi-angles $\delta_{1,2}$ (fig. AEV-T.2.3.1.1) and the angle between the axes $\Sigma = \delta_1 + \delta_2$ which can be considered a generalization of the cylindrical gear which has the point O at infinity. Thus, the elements of the bevel gear can be studied by analogy with those of the cylindrical gear considering the correlations: <u>circles in the plane become circles on the sphere, the plane involute becomes the spherical involute, the gear line becomes a diametrical circle, the reference rack becomes a reference plane wheel</u>. - In the case of bevel gears being made with straight-profile tools (straight-line cutting edges), an approximate involute profile is obtained with the diametrical gear circle degenerated into an <u>octoid</u>, hence the name <u>octoidal gear</u>. - To define the geometry of the bevel gear, by analogy with the cylindrical gear, the <u>spherical surfaces are approximated with frontal conical surfaces</u> (developable in the plane) tangent to the spheres (Tredgold approximation). - The bevel gear, being an approximate involute gear, can only be undisplaced or <u>zero-displaced</u> with radial and/or tangential displacements. - The geometric parameters for representation in the execution drawings (with technological implications) are defined at the level of the <u>outer front cone</u> (index e, which may be missing in the technical documentation), and the geometric parameters for calculating forces and strength are defined at the level of the <u>middle front cone</u> (index m, fig. AEV-T.2.3.1.2).	- <u>Diameters of average pitch circles</u> (fig. AEV-T.2.3.1.2; 1 - pinion; 2 - bevel wheel), $d_{m1} = d_{e1}(1 - \psi_d \sin \delta_1) = d_{e1}(1 - 0,5 \psi_R)$ (AEV-T.2.3.1.2) $d_{m1,2} = 2 R_m \sin \delta_{1,2}$ (AEV-T.2.3.1.3) with $\psi_d = b/d_{e1}$ the width factor related to the pinion diameter, $\psi_R = b/R_e$ – the width factor related to the generator length, d_{e1} – the diameter of the outer pitch circle of the pinion, R_m – the length of the generator of the middle pitch cone, R_e – the length of the generator of the outer pitch cone, $\delta_{1,2}$ – the semi-angles of the pitch cones, b – the width of the teeth. - <u>The average module</u>, considering in (AEV-T.2.3.1.2) $d_{m1} = m_m z_1$ and $d_{e1} = m_e z_1$, $m_m = m_e (1 - \psi_d \sin \delta_1) = m_e (1 - 0,5 \psi_R)$ (AEV-T.2.3.1.3) - <u>Gear ratio</u>, $u = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{d_{e2}}{d_{e1}} = \frac{z_2}{z_1} = \frac{\sin \delta_2}{\sin \delta_1};$ (AEV-T.2.3.1.1) for the <u>orthogonal bevel gear</u> with $\Sigma = \delta_1 + \delta_2 = 90^\circ$, $u = 1/\tan \delta_1 = \tan \delta_2$
Note. a. <u>Spur-toothed</u> bevel gears, for reasons of vibration- and noise-free operation, are used for low peripheral speeds ($v = 2 \dots 3$ m/s). b. Bevel gears with <u>inclined teeth</u> operate at peripheral speeds of up to 12 m/s, are made on the same machines on which spur gears are made, under conditions of much reduced productivity and, therefore, are rarely used in practice. c. Bevel gears with <u>curved teeth</u> operate under conditions of increased silence and durability at high peripheral speeds (up to 12 m/s), are manufactured on specialized high-productivity machines and have geometric parameters dependent on the technological toothing processes, the type of machine and toothing tool [Velicu, 2004].	

- the curved arc toothing (Gleason) is performed with a knife-holder head (fig. AEV-T.2.3.1.3,a) equipped with cutting blades with rectilinear edges, arranged in a circle; the contact between the flanks of the wheel teeth is theoretically point-like; the median inclination angle of the arc toothing, β_m (usually, 35°), is dependent on the parameters of the toothing tools.

- the eloid curved toothing, usually based on the Oerlikon-Spiromatic process, has a plane-wheel tooth flank line shaped as an elongated epicycloid, obtained by using a knife holder head that has the knives fixed on different spirals to obtain the median inclination angle of the tooth, $\beta_m = 35^\circ$ (fig. AEV-T.2.3.1.3,b).

d. Crossed-axis (hypoid) bevel gears have hyperboloidal pitch surfaces (rolling surfaces) of rotation and have a constant transmission ratio. The teeth of hypoid wheels are curved (fig. AEV-T.2.3.1.4) and are machined on the same gearing machines as for the wheels of intersecting-axis (concurrent) gears.



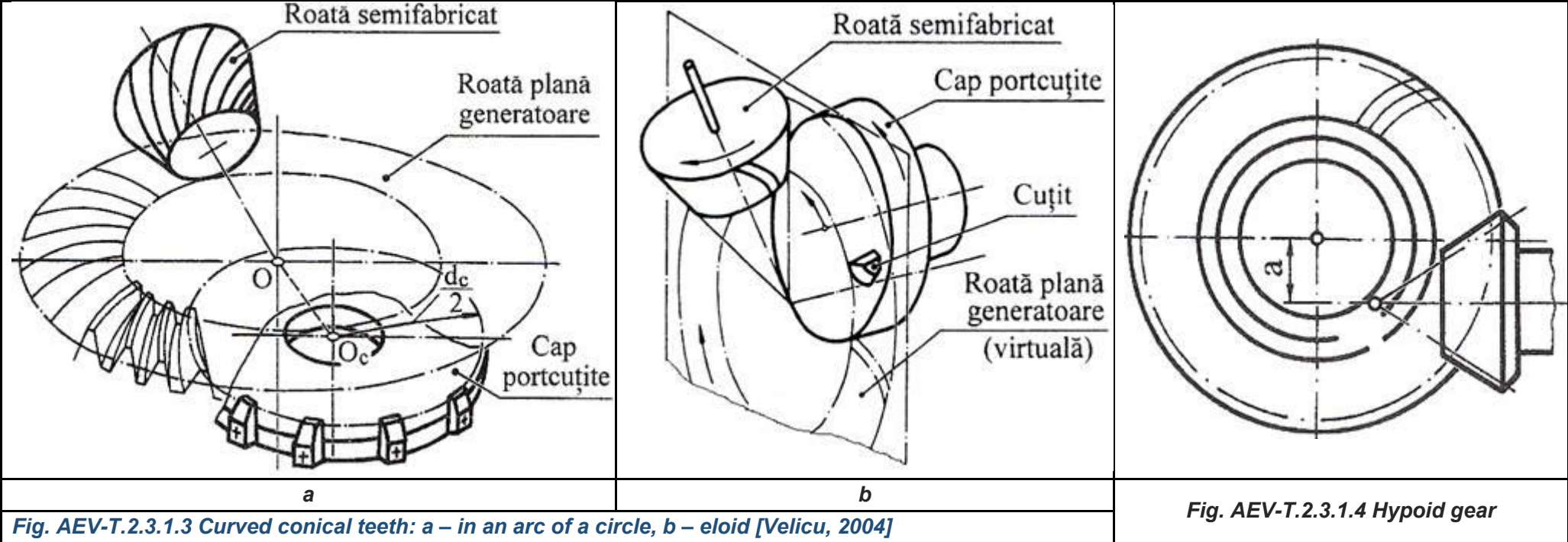
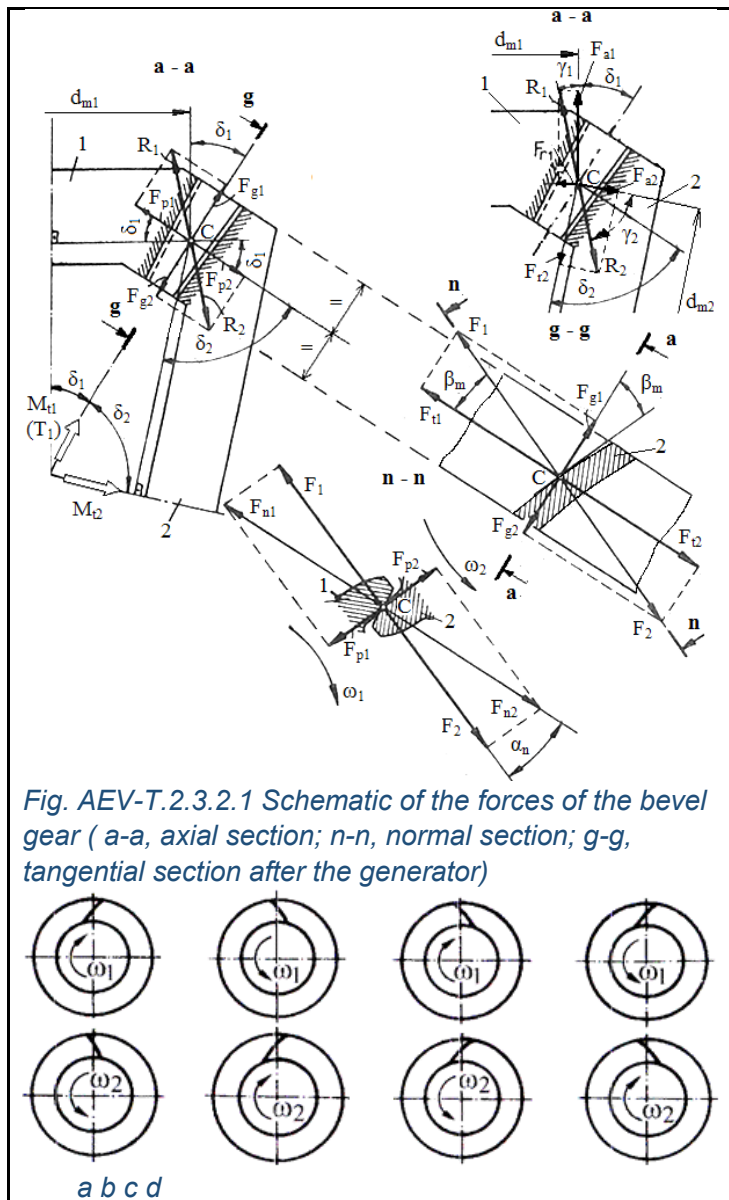


Fig. AEV-T.2.3.1.3 Curved conical teeth: a – in an arc of a circle, b – eloid [Velicu, 2004]

AEV-T.2.3.2 MODELING OF BEVEL GEAR LOADS [Rădulescu, 1985]

	<i>Simplifying assumptions:</i> - normal forces are considered applied to the gear pole C associated with the middle front cone, - friction is neglected, - forces are considered to be applied statically. <i>Tab. AEV-T.2.3.2.1 Determination of forces</i> <table><tr><th data-bbox="819 1166 1823 1238"><i>Parameter and calculation relationship</i></th><th data-bbox="1823 1166 2143 1238"><i>Direction and meaning</i></th></tr></table>	<i>Parameter and calculation relationship</i>	<i>Direction and meaning</i>
<i>Parameter and calculation relationship</i>	<i>Direction and meaning</i>		



Tangential forces,

$$F_{t1} = F_{t2} = F_t = \frac{2 T_1}{d_{m1}} \quad (\text{AEV-T.2.3.2.1})$$

Direction tangent to the pitch circle; opposite direction to speed (resisting force), for the driving wheel, and same direction as speed (driving force), for the driven wheel

Radial forces,

$$\begin{aligned} F_{r1} &= R_1 \sin \gamma_1 = F_{p1} \cos \delta_1 \mp F_{g1} \sin \delta_1 = \\ &= F_1 \operatorname{tg} \alpha_n \cos \delta_1 \mp F_{g1} \sin \delta_1 = \frac{F_{t1}}{\cos \beta_m} \operatorname{tg} \alpha_n \cos \delta_1 \mp F_{t1} \operatorname{tg} \beta_m \sin \delta_1 = \\ &= \frac{2 T_1}{d_{m1} \cos \beta_m} (\operatorname{tg} \alpha_n \cos \delta_1 \mp \sin \beta_m \sin \delta_1) \quad (\text{AEV-T.2.3.2.2}) \end{aligned}$$

Radial direction; direction towards the wheel axis

$$\begin{aligned} F_{r2} &= R_2 \sin \gamma_2 = F_{p2} \cos \delta_2 \pm F_{g2} \sin \delta_2 = \\ &= F_2 \operatorname{tg} \alpha_n \cos \delta_2 \pm F_{g2} \sin \delta_2 = \frac{F_{t2}}{\cos \beta_m} \operatorname{tg} \alpha_n \cos \delta_2 \pm F_{t2} \operatorname{tg} \beta_m \sin \delta_2 = \\ &= \frac{2 T_1}{d_{m1} \cos \beta_m} (\operatorname{tg} \alpha_n \cos \delta_2 \pm \sin \beta_m \sin \delta_1) \quad (\text{AEV-T.2.3.2.3}) \end{aligned}$$

Axial forces,

$$\begin{aligned} F_{a1} &= R_1 \cos \gamma_1 = F_{p1} \sin \delta_1 \pm F_{g1} \cos \delta_1 = \\ &= F_1 \operatorname{tg} \alpha_n \sin \delta_1 \mp F_{g1} \sin \delta_1 = \frac{F_{t1}}{\cos \beta_m} \operatorname{tg} \alpha_n \cos \delta_1 \pm F_{t1} \operatorname{tg} \beta_m \cos \delta_1 = \\ &= \frac{2 T_1}{d_{m1} \cos \beta_m} (\operatorname{tg} \alpha_n \sin \delta_1 \pm \sin \beta_m \cos \delta_1) \quad (\text{AEV-T.2.3.2.4}) \end{aligned}$$

Axial direction; outward direction

$$\begin{aligned} F_{a2} &= R_2 \cos \gamma_2 = F_{p2} \sin \delta_2 \mp F_{g2} \cos \delta_2 = \\ &= F_2 \operatorname{tg} \alpha_n \sin \delta_2 \mp F_{g2} \sin \delta_2 = \frac{F_{t2}}{\cos \beta_m} \operatorname{tg} \alpha_n \cos \delta_2 \mp F_{t2} \operatorname{tg} \beta_m \cos \delta_2 = \\ &= \frac{2 T_1}{d_{m1} \cos \beta_m} (\operatorname{tg} \alpha_n \sin \delta_2 \mp \sin \beta_m \cos \delta_2) \quad (\text{AEV-T.2.3.2.5}) \end{aligned}$$

Fig. AEV-T.2.3.2.2 The directions of rotation of the wheels (pinion) and the directions of inclination of its teeth: a – clockwise rotation, the direction of inclination of the tooth to the right; b – counterclockwise rotation, the direction of inclination of the tooth to the left; c – clockwise rotation, the direction of inclination of the tooth to the left; b – counterclockwise rotation, the direction of inclination of the tooth to the right

Normal force,

$$F_n = \frac{F_t}{\cos \beta_m \cos \alpha_n} = \frac{2 T_1}{d_{m1} \cos \beta_m \cos \alpha_r} \quad (\text{AEV-T.2.3.2.6})$$

Direction according to the common normal of the profiles in contact; opposite direction to the speed, for the driving wheel, and the same direction as the speed, for the driven wheel

Note.

a. Meanings of geometric parameters in fig. AEV-T.2.3.2.1: $\delta_{1,2}$ – semi-angles of the dividing cones (rolling); $d_{m1,2}$ – diameters of the average dividing circles; β_m – angle of inclination of the teeth on the average dividing circle; α_n – angle of engagement in the median normal plane; $\omega_{1,2}$ – angular velocities; M_{t1} (T_1) – torque of the bevel pinion; M_{t2} – torque of the bevel wheel

b. In the relations (AEV-T.2.3.2.2)...(AEV-T.2.3.2.5): the upper sign is for the situation in which the direction of rotation of the pinion is the same as the direction of inclination of its teeth (viewed from the top of the cone, fig. AEV-T.2.3.2.2, a,d); the lower sign is for the case in which the direction of rotation of the pinion is opposite to the direction of inclination of its teeth (viewed from the top of the cone), fig. AEV-T.2.3.2.2, b,c. In fig. AEV-T.2.3.2.1 the force diagram for the first situation (same directions) is represented, the direction of rotation of the pinion is clockwise and the direction of inclination of its tooth is to the right;

c. Particular cases:

Non-orthogonal bevel gear with straight teeth ($\beta_m = 0$; $\alpha_n = \alpha$)

$$F_t = \frac{2 T_1}{d_{m1}}$$

$$F_{r1,2} = \frac{2 T_1}{d_{m1}} \operatorname{tg} \alpha \cos \delta_{1,2}$$

$$F_{a1,2} = \frac{2 T_1}{d_{m1}} \operatorname{tg} \alpha \sin \delta_{1,2}$$

$$F_n = \frac{2 T_1}{d_{m1} \cos \alpha}$$

$$F_{t1} = F_{t2} = F_t; F_{n1} = F_{n2} = F_n$$

Orthogonal bevel gear with straight teeth ($\beta_m = 0$; $\alpha_n = \alpha$; $\delta_1 + \delta_2 = 90^\circ$; Fig. AEV-T.2.3.2.3)

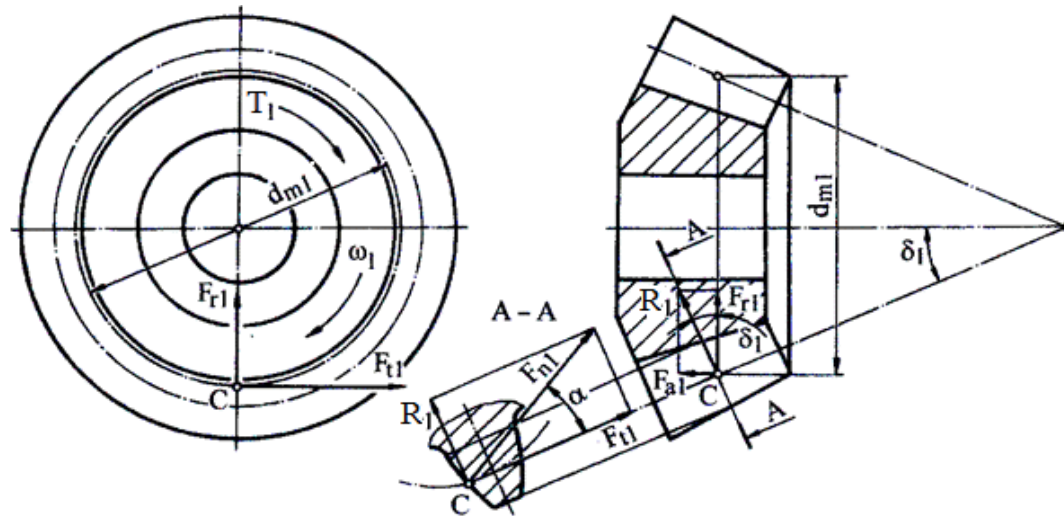
$$F_t = \frac{2 T_1}{d_{m1}}$$

$$F_{r1} = R_1 \cos \delta_1 = F_{t1} \operatorname{tg} \alpha \cos \delta_1 = \frac{2 T_1}{d_{m1}} \operatorname{tg} \alpha \cos \delta_1$$

$$F_{a1} = R_1 \sin \delta_1 = F_{t1} \operatorname{tg} \alpha \sin \delta_1 = \frac{2 T_1}{d_{m1}} \operatorname{tg} \alpha \sin \delta_1$$

$$F_n = \frac{2 T_1}{d_{m1} \cos \alpha}$$

$$F_{t1} = F_{t2} = F_t; F_{r1} = F_{a2}; F_{a1} = F_{r2}; F_{n1} = F_{n2} = F_n$$



a b

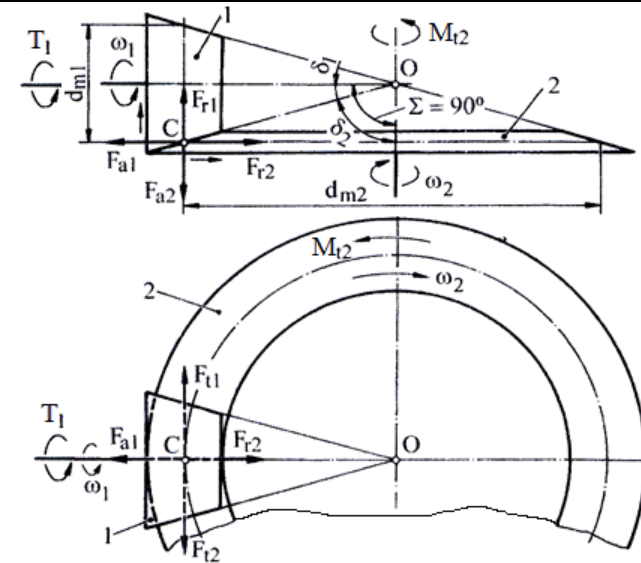


Fig. AEV-T.2.3.2.3 Forces of the orthogonal bevel gear with straight teeth: a – pinion loading diagram, b – gear force diagram

AEV-T.2.3.3 CALCULATION MODEL OF CONTACT STRESS IN BEVEL GEARS [Jula, 1989; Rădulescu, 1985; Moldovean, 2001]

Tab. AEV-T.2.3.3.1 Definition and parameters of the virtual equivalent cylindrical gear

Definition of virtual equivalent cylindrical gear and equivalent wheel (fig. AEV-T.2.3.3.1):

The strength calculation of the bevel gear with curved (inclined) teeth, considering that the interaction force F_n acts in the median normal plane, is performed in the median plane of an equivalent cylindrical gear (virtual, hence the index v) with inclined teeth.

The equivalent wheel of the equivalent helical gear has teeth with the same shape and dimensions as the teeth of the real bevel gear in the median normal plane; for mathematical equalities will be considered:

- the normal module of the equivalent wheels is equal to the median normal module of conical wheels;
- the angle of inclination of the teeth of the equivalent wheels is equal to the median division angle of inclination of the teeth of the bevel wheels;
- the radii of the division circles of the equivalent wheels are equal to the lengths of the generators of the median front cones of the conical wheels;
- the height of the teeth of the replacement wheels is equal to the height of the teeth of the bevel wheels in the median section

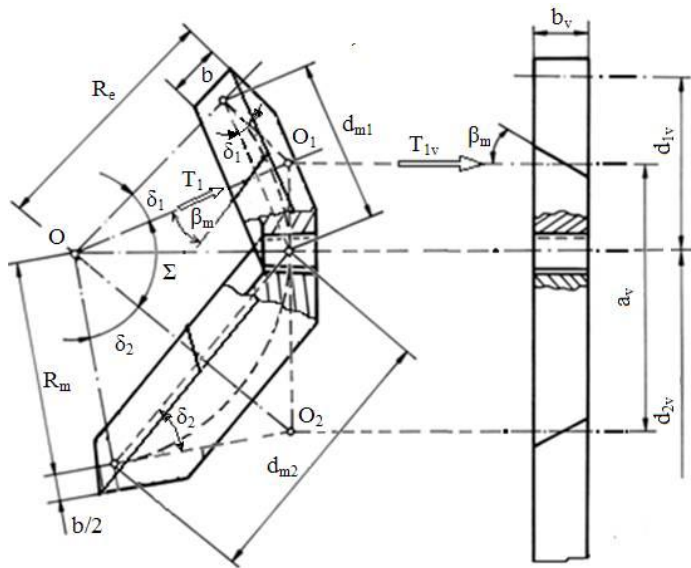


Fig. AEV-T.2.3.3.1 Equivalence model of a bevel gear with helical or curved teeth with a cylindrical gear with helical teeth

Equivalent cylindrical gear parameters:

Diameters of the division circles,

$$d_{1,2v} = \frac{d_{m1,2}}{\cos \delta_{1,2}} \quad (\text{AEV-T.2.3.3.1})$$

The numbers of teeth,

Substituting in the relation (AEV-T.2.3.3.1), $d_{1,2v} = m_m Z_{1,2v}$, $d_{m1,2} = m_m Z_{1,2}$ (the normal module of the equivalent wheel is equal to the average normal module), it results,

$$Z_{1,2v} = \frac{Z_{1,2}}{\cos \delta_{1,2}} \quad (\text{AEV-T.2.3.3.2})$$

Gear ratio,

Considering rel. (AEV-T.2.3.3.2),

$$u_n = \frac{Z_{2v}}{Z_{1v}} = u \frac{\cos \delta_1}{\cos \delta_2} \quad (\text{AEV-T.2.3.3.3})$$

The distance between the axes,

Taking into account the relation (AEV-T.2.3.3.2) and, it results,

$$a_v = \frac{d_{1v} + d_{2v}}{2} = \frac{1}{2} \left(\frac{d_{m1}}{\cos \delta_1} + \frac{d_{m2}}{\cos \delta_2} \right) = R_m (\operatorname{tg} \delta_1 + \operatorname{tg} \delta_2) \quad (\text{AEV-T.2.3.3.4})$$

The torque,

Taking into account the relation (AEV-T.2.3.3.2) it results,

$$T_{1v} = F_t \frac{d_{1v}}{2} = F_t \frac{d_{m1}}{2 \cos \delta_1} = T_1 \frac{1}{\cos \delta_1} \quad (\text{AEV-T.2.3.3.5})$$

Tooth width (fig. AEV-T.2.3.3.1),

$$b = b_v \quad (\text{AEV-T.2.3.3.6})$$

The contact calculation of the bevel gear with curved teeth is done through the equivalent helical gear.

The contact calculation relations (tab. AEV-T.2.3.3.2) are obtained by replacing the parameters of the equivalent gear (tab. AEV-2.3.3.1) in the contact calculation relations of the cylindrical gear with helical teeth from chap. AEV-T.2.2, tab. [AEV-T.2.2.3.2](#).

Tab. AEV-T.2.3.3.2 Calculation relations for contact of bevel gears

Contact stress	sizing	Verification
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General case. Non-orthogonal bevel gear with curved (inclined) teeth ($\Sigma \neq 90, \beta_m \neq 0$)		$\sigma_H \leq \sigma_{HP1,2}, \quad (\text{AEV-T.2.3.3.15})$ $\sigma_{HP1,2} = \frac{\sigma_{H \lim} Z_{N1,2}}{S_{H \min}} Z_L Z_R Z_v Z_w$, $\sigma_{HP1,2}$ – <u>permissible contact stresses for pinion and wheel</u> ; $\sigma_{H \lim}$ – <u>limit contact stress</u> (fer. AEV-M.1); $S_{H \min}$ – <u>minimum safety factor for contact</u> (fer. AEV-F.1); $Z_{N1,2}$ – <u>factors of the number of load cycles of the pinion and the wheel</u> (fer. AEV-F.13); Z_R – <u>flank roughness factor</u> (fer. AEV-F.11); Z_L – <u>lubricant factor</u> (fer. AEV-F.12); Z_v – <u>speed factor</u> (fer. AEV-F.10); Z_w – <u>hardness factor</u> (fer. AEV-F.9); Z_x – <u>size factor</u> (fer. AEV-F.8)
$\sigma_H = Z_E Z_\epsilon Z_H Z_\beta \sqrt{\frac{2T_1}{b d_{e1}^2} \frac{K_A K_V K_{H\beta} K_{H\alpha}}{(1-\psi_d \sin \delta_1)^2 u \sin \delta_1}} \quad (\text{AEV-T.2.3.3.7})$	$d_{e1} = \sqrt[3]{\frac{2T_1}{\psi_d (1-\psi_d \sin \delta_1)^2} \frac{K_A K_V K_{H\beta} K_{H\alpha}}{\sigma_{HP}} \left(\frac{Z_E Z_\epsilon Z_H Z_\beta}{\sigma_{HP}} \right)^2} \quad (\text{AEV-T.2.3.3.8})$	
Particular case 1. Orthogonal bevel gear with curved (inclined) teeth ($\Sigma = 90, \beta_m \neq 0$)		
$\sigma_H = Z_E Z_\epsilon Z_H Z_\beta \sqrt{\frac{2T_1}{b d_{e1}^2} \frac{K_A K_V K_{H\beta} K_{H\alpha}}{(\sqrt{1+u^2}-\psi_d)^2}} (1+u^2) \quad (\text{AEV-T.2.3.3.9})$	$d_{e1} = \sqrt{1+u^2} \sqrt[3]{\frac{2T_1}{\psi_d (\sqrt{1+u^2}-\psi_d)^2} \frac{K_A K_V K_{H\beta} K_{H\alpha}}{\sigma_{HP}} \left(\frac{Z_E Z_\epsilon Z_H}{\sigma_{HP}} \right)^2} \quad (\text{AEV-T.2.3.3.10})$	
Particular case 2. Non-orthogonal bevel gear with straight teeth ($\Sigma \neq 90, \beta_m = 0$)		
$\sigma_H = Z_E Z_H \sqrt{\frac{2T_1}{b d_{e1}^2} \frac{K_A K_V K_{H\beta}}{(1-\psi_d \sin \delta_1)^2 u \sin \delta_1} \frac{\sin \Sigma}{1}} \quad (\text{AEV-T.2.3.3.11})$	$d_{e1} = \sqrt[3]{\frac{2T_1}{\psi_d (1-\psi_d \sin \delta_1)^2} \left(\frac{Z_E Z_H}{\sigma_{HP}} \right)^2 \frac{\sin \Sigma}{u \sin \delta_1}} \quad (\text{AEV-T.2.3.3.12})$	
Particular case 3. Orthogonal bevel gear with straight teeth ($\Sigma = 90, \beta_m = 0$)		
$\sigma_H = Z_E Z_H \sqrt{\frac{2T_1}{b d_{e1}^2} \frac{K_A K_V K_{H\beta} K_{H\alpha}}{(\sqrt{1+u^2}-\psi_d)^2} \frac{(1+u^2)^{3/2}}{u}} \quad (\text{AEV-T.2.3.3.13})$	$d_{e1} = \sqrt{1+u^2} \sqrt[3]{\frac{2T_1}{\psi_d (\sqrt{1+u^2}-\psi_d)^2} \frac{K_A K_V K_{H\beta}}{\sigma_{HP}} \left(\frac{Z_E Z_H}{\sigma_{HP}} \right)^2} \quad (\text{AEV-T.2.3.3.14})$	
Note		
a. z_1 is the number of teeth of the pinion(fer. AEV-P.1.1); $Z_H = \sqrt{\frac{2}{\cos \alpha \sin \alpha}}$ - contact area factor (fer. AEV-F.7); $Z_\beta = \sqrt{\cos \beta_m}$ - tooth inclination factor for contact stress. b. The contact stresses, mentioned in column 1, as well as the dimensioning relations, in column 2, can be determined as a function of the external module, m_e, or of the external generator length, R_e, as a result of using the relation $d_{e1} = m_e z_1$ or, respectively, $d_{e1} = 2 R_e \sin \delta_1$. In addition, they can also be expressed as dependent on the generator-dependent width factor by replacing the diametrical width factor ψ_d with the relation, $\psi_d = \frac{\psi_R}{2 \sin \delta_1}$.		

AEV-T.2.3.4 MODEL FOR CALCULATING BENDING STRESSES IN BEVEL GEARS

The bending calculation of the bevel gear with curved teeth is also done through the equivalent helical gear.

The bending calculation relations (tab. AEV-T.2.3.4.1) are obtained by replacing the equivalent gear parameters (tab. AEV-2.3.3.1) in the contact calculation relations of the helical gear from chap. AEV-T.2.2 tab. [AEV-T.2.2.4.1](#).

Tab. AEV-T.2.3.4.1 Calculation relations for bending for bevel gear

Maximum bending stresses (1 – pinion, 2- wheel)		sizing		Verification	
General case. Non-orthogonal bevel gear with curved (inclined) teeth ($\Sigma \neq 90, \beta_m \neq 0$)					
$\sigma_{F1,2} = \frac{2 T_1 z_1 K_A K_V K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta}{b_{1,2} d_{e1}^2 (1-\psi_d \sin \delta_1)^2 \cos \beta_m} Y_{Sa1,2} Y_F$	(AE) V- T. 2.3. 4.1)	$d_{e1} = \sqrt[3]{\frac{2 T_1 z_1 K_A K_V K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta}{\psi_d (1-\psi_d \sin \delta_1)^2 \cos \beta_m} \left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max}}$	(AE) V- T. 2.3. 4.2)	$\sigma_{F1} \leq \sigma_{FP1}, \sigma_{F2} \leq \sigma_{FP2};$ $\sigma_{FP1,2} = \frac{\sigma_{F \lim 1,2} Y_{N1,2} Y_{ST}}{S_{F \min}} Y_{\delta 1,2}$ $\sigma_{FP1,2}$ – permissible bending stresses for pinion and wheel; $\sigma_{F \lim 1,2}$ – limit bending stress for pinion and wheel (fer. AEV-M.1); $S_{F \min}$ – minimum safety factor for bending (fer. AEV-F.1); $Y_{N1,2}$ – factor of the number of load cycles of the pinion and the wheel (fer. AEV-F.13), Y_{ST} – reference bending stress factor ($Y_{ST} = 2$); $Y_{\delta 1,2}$ – stress concentrator factors of the pinion and wheel (fer. AEV-F.15); $Y_{R1,2}$ – roughness factors of the pinion and wheel fillets (fer. AEV-F.11), Y_x – size factor for bending (fer. AEV-F.8)	
Particular case 1. Orthogonal bevel gear with curved (inclined) teeth ($\Sigma = 90, \beta_m \neq 0$)					
$\sigma_{F1,2} = \frac{2 T_1 z_1 K_A K_V K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta}{b_{1,2} d_{e1}^2 \cos \beta_m} \frac{\sqrt{1+u^2}}{(\sqrt{1+u^2}-\psi_d)^2} Y_{Sa1,2} Y_F$	(AE) V- T. 2.3. 4.3)	$d_{e1} = \sqrt[3]{\frac{2 T_1 z_1 K_A K_V K_{F\beta} K_{F\alpha} Y_\epsilon Y_\beta}{\psi_d \cos \beta_m} \frac{\sqrt{1+u^2}}{(\sqrt{1+u^2}-\psi_d)^2} \left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max}}$	(AE) V- T. 2.3. 4.4)		
Particular case 2. Non-orthogonal bevel gear with straight teeth ($\Sigma \neq 90, \beta_m = 0$)					
$\sigma_{F1,2} = \frac{2 T_1 z_1 K_A K_V K_{F\beta}}{b_{1,2} d_{e1}^2 (1-\psi_d \sin \delta_1)^2} Y_{Sa1,2} Y_{Fa1,2}$	(AE) V- T. 2.3. 4.5)	$d_{e1} = \sqrt[3]{\frac{2 T_1 z_1 K_A K_V K_{F\beta}}{\psi_d (1-\psi_d \sin \delta_1)^2} \left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max}}$	(AE) V- T. 2.3. 4.6)		
Particular case 3. Orthogonal bevel gear with straight teeth ($\Sigma = 90, \beta_m = 0$)					
$\sigma_{F1,2} = \frac{2 T_1 z_1 K_A K_V K_{F\beta}}{b_{1,2} d_{e1}^2} \frac{\sqrt{1+u^2}}{(\sqrt{1+u^2}-\psi_d)^2} Y_{Sa1,2} Y_F$	(AE) V- T. 2.3. 4.7)	$d_{e1} = \sqrt[3]{\frac{2 T_1 z_1 K_A K_V K_{F\beta}}{\psi_d} \frac{\sqrt{1+u^2}}{(\sqrt{1+u^2}-\psi_d)^2} \left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max}}$	(AE) V- T. 2.3. 4.8)		

Note.

- a. z_1 is the number of teeth of the pinion (fer. [AEV-P.1.1](#)); $\left(\frac{Y_{Sa} Y_{Fa}}{\sigma_{FP}} \right)_{\max} = \max \left(\frac{Y_{Sa1} Y_{Fa1}}{\sigma_{FP1}}, \frac{Y_{Sa2} Y_{Fa2}}{\sigma_{FP2}} \right)$; Y_β – the contact line inclination factor (fer. [AEV-F.9](#)).
- b. The contact stresses, mentioned in column 1, as well as the dimensioning relations in column 2, can be expressed as a function of the external module, m_e , or of the external generator length, R_e , as a result of using the relation $d_{e1} = m_e z_1$ or, respectively, $d_{e1} = 2 R_e \sin \delta_1$.

AEV-T.2.3.5 CALCULATION SPECIFICATIONS OF HYPOID GEARS

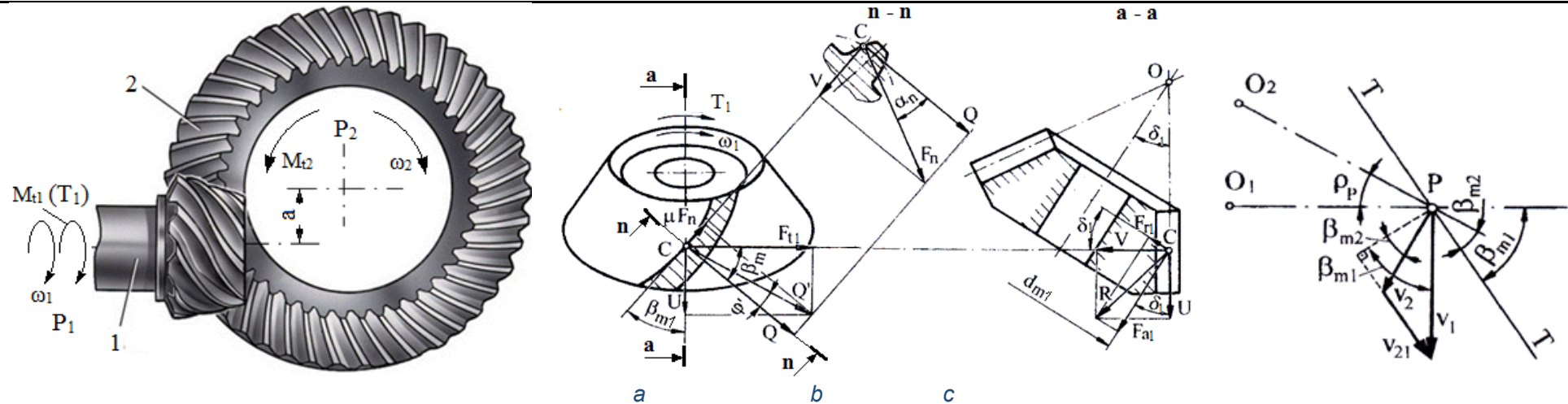


Fig. AEV-T.2.3.5.5 Hypoid gear: a – general diagram, b – pinion force diagram [Moldoveanu, 2001]; c – speed diagram [Moldoveanu, 2001]

Geometric, kineto-static, constructive and technological particularities:

- hypoid gears are gears with crossed axes, only with curved teeth;
- the flanks of the teeth of the wheels of a hypoid gear can be generated by meshing these wheels with an imaginary flat wheel described by the cutting edges of the gearing tools, analogous to the gearing of bevel gears with concurrent axes (they are processed on the same gearing machines for bevel gears);
- the number of teeth of the pinion $z_1 = 5 \dots 15$, depending on the value of the gear ratio (for small values of the gear ratio, large values are adopted and vice versa);
- the angle of the dividing cone δ_2 of the driven wheel has the value $\delta_2 = 67^\circ \dots 70^\circ$, usually, 70° ;
- the angle of inclination of the wheel teeth $\beta_{m2} \leq 35^\circ$, usually values close to 35° are used, and for the pinion, $\beta_{m1} \leq 50^\circ$, usually values close to 50° are used;
- the main geometric elements have values conditioned both by the toothing process and by the tool parameters for cutting the teeth;
- these gears are used, mainly, in the central transmissions of automobiles where, as a consequence of the distance between the axles, they lead to an increase in their passage capacity (when the wheel axis is located below the pinion axis) or to an increase in the interior space (when the pinion axis is located below the wheel axis).

- due to the relatively large slippage of the flanks of the teeth in engagement, along them, an additional friction force μF_n appears, acting along the common tangent to the flank lines;
- as a result of the sliding friction along the teeth (fig. AEV-T.2.3.5.5,b), taking into account the relationship (AEV-T.2.3.5.1), the torque at the wheel,

$$M_{t2} = \frac{P_2}{\omega_2} = \frac{P_1}{\omega_1} u \eta = T_1 u \eta = T_1 \frac{d_{m2}}{d_{m1}} \frac{\cos \beta_{m2}}{\cos \beta_{m1}} \eta, \quad (\text{AEV-T.2.3.5.5})$$

where, T_1 is the pinion torque, η - the efficiency of the hypoid gear; the ratio $\cos \beta_{m2} / \cos \beta_{m1} \cong 1.2 \dots 1.3$ highlights the increase in the dimensions (rigidity) of the pinion;

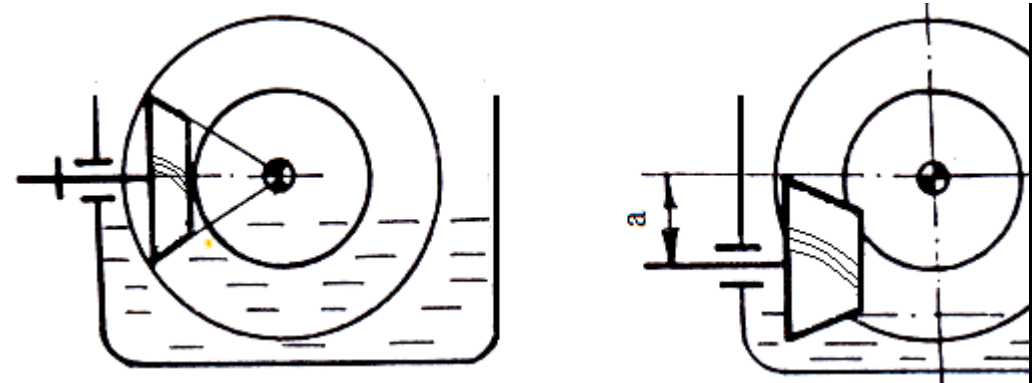
- in practice, the hypoid gear is frequently used in which the pinion axis is moved in the same direction as the inclination of the wheel teeth, a solution that ensures a more rigid pinion construction than that of the intersecting-axis bevel gear because the frontal module and the pitch diameter increase;
- the strength calculation (contact and bending) is based on the cylindrical gear with helical teeth equivalent to the hypoid gear, similar to the intersecting-axis bevel gear with curved teeth;
- splash lubrication of the hypoid gear involves a smaller volume of oil and reduced friction between the wheels and the lubricant (fig. AEV-T.2.3.5.6).

- the gear ratio, taking into account that $v_1 \cos \beta_{m1} = v_2 \cos \beta_{m2}$ (fig. AEV-T.2.3.5.5,c) is determined by the relationship,

$$u = \frac{\omega_1}{\omega_2} = \frac{2v_1/d_{m1}}{2v_2/d_{m2}} = \frac{d_{m2} \cos \beta_{m2}}{d_{m1} \cos \beta_{m1}} = \frac{m_{mt2} z_2 \cos \beta_{m2}}{m_{mt1} z_1 \cos \beta_{m1}} =$$

$$= \frac{m_{mn2} / \cos \beta_{m2}}{m_{mn1} / \cos \beta_{m1}} \frac{\cos \beta_{m2}}{\cos \beta_{m1}} = \frac{z_2}{z_1}, \quad (\text{AEV-T.2.3.5.5,c})$$

where, $\omega_{1,2}$ [rad/s] is the angular velocity of the pinion, respectively the wheel;
 $v_{1,2}$ [mm/s] - the peripheral velocity of the contact point for the pinion, respectively the wheel;
 $d_{m1,2}$ [mm] - the diameters of the median pitch circles; $\beta_{m1,2}$ [°] - the median inclination angle of the pinion tooth, respectively, of the wheel;
 $m_{mt1,2}$ [mm] - the median frontal module;
 $m_{mn1,2}$ [mm] - the median normal module ($m_{mn1} = m_{mn2} = m_{mn}$, meshing condition); $z_{1,2}$ - the number of teeth of the pinion, respectively, of the wheel.



a b
 Fig. AEV-T.2.3.5.6 Lubrication of gears with curved teeth: a – orthogonal bevel, b – hypoid

AEV-T.2.4 MODELING OF CYLINDRICAL WORM GEARS

AEV-T.2.4.1 GEOMETRIC, FUNCTIONAL AND CONSTRUCTIVE PARTICULARITIES OF WORM GEARS [Rădulescu, 1985; Moldovean, 2001]

Geometric and constructive features

Main kinematic and geometric parameters

- From the point of view of the spatial position of the axes, the worm gear is a helical gear (fig. AEV-T.2.4.1.1, fig. AEV-T.2.4.1.3,a), which has the distance between the axes a and the angle between them $\Sigma = \beta_1 + \beta_2 = 90^\circ$.

- The cylindrical worm gear has wheel 1 (fig. AEV-T.2.4.1.2,a; fig. AEV-T.2.4.1.3,b) called, worm, with a small number of teeth (starts, $z_1 = 1 \dots 4$) and a large angle of inclination of the teeth $\beta_1 = 90^\circ - \gamma$, and the conjugate wheel, called worm wheel, is a toroidal gear that wraps around the worm with the angle of inclination $\beta_2 = \gamma$.

- The globoidal worm gear, in addition to the cylindrical worm gear, has the worm with a globoidal shape (it covers the worm wheel, fig. AEV-T.2.4.1.3, a; fig. AEV-T.2.4.1.3, c)

- Due to the toroidal shape of the worm wheel, the teeth of the worm gear wheels can no longer be determined by a reference rack but by a standardized reference worm; six types of reference worms are standardized, depending on the processing process, of which the most used is the Archimedean worm which, in axial section, has a rectilinear tooth flank (fig. AEV-T.2.4.1.2,a).

- The defining parameters of the standardized reference worms are established with respect to the reference cylinder on which the tooth thicknesses are equal to the widths of the spaces between the teeth (fig. AEV-T.2.4.1.2,a).

- The standardized module is the axial module of the worm (the front module of the wheel) $m_x = m_{x1} = m_{t2}$; the normal module, $m_n = m_x \cos \gamma$; the front module, $m_{t1} = m_x \operatorname{ctg} \gamma$ (fig. AEV-T.2.4.1.2,a).

- The profile displacements of the wheel teeth are made for the same reasons as for cylindrical gears, with the difference that they are made only for the worm.

- Gear ratio (transmission),

$$u = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{z_2}{z_1} = \frac{d_2}{d_1} \operatorname{ctg} \gamma, \quad (\text{AEV-T.2.4.1.1})$$

where, $d_{1,2}$ are the diameters of the division circles of the worm, respectively, the worm wheel; γ - the lead angle of the worm; $\omega_{1,2} = v_{1,2} / (d_{1,2}/2)$ - the angular velocities of the worm, respectively, the wheel; $v_{1,2}$ - the peripheral velocities of the worm, respectively, the worm wheel in the gearing pole (from Fig. AEV-T.2.4.2.1, $v_2 = v_1 \operatorname{ctg} \gamma$); $n_{1,2}$ - the revolutions of the worm, respectively, the worm wheel; $z_{1,2}$ - the numbers of teeth (starts) of the worm, respectively, the worm wheel.

- The lead angle of the worm (helix angle),

$$\gamma = \operatorname{artg} \frac{p_{x1}}{p_{t1}} = \operatorname{artg} \frac{\pi m_{x1}}{\pi m_{t1}} = \operatorname{artg} \frac{m_{x1}}{d_1} = \operatorname{artg} \frac{m_{x1} z_1}{d_1} = \operatorname{artg} \frac{z_1}{q} \quad (\text{AEV-T.2.4.1.2})$$

where, d_1 is the pitch diameter equal to the diameter of the worm's reference cylinder for the gear with unshifted teeth ($d_1 = d_{01} = m_{x1} q$); $p_{x1} = \pi m_{x1}$ - axial pitch of the worm; $m_{x1} = m_x$ - axial module of the worm (standard values); $q = z_1 / \operatorname{tg} \gamma$ - diametrical factor of the worm with standard values depending on the axial module m_x (e.g., $q = 9, 10, 11$ or 12 for $m_x = 3 \dots 8$ mm).

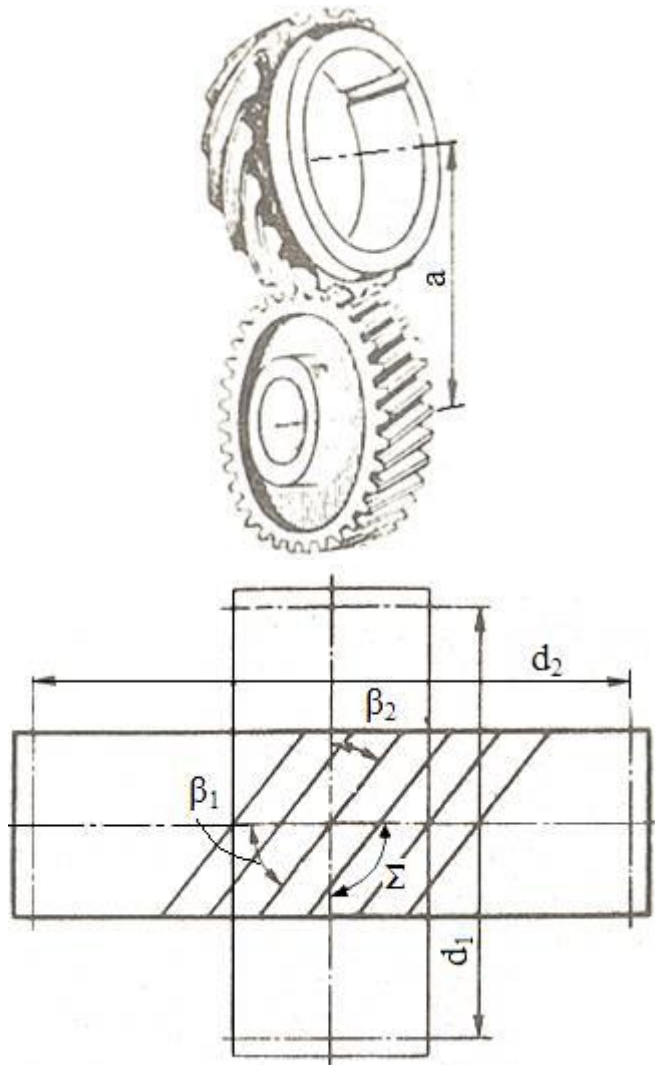
- Diameters of the pitch circles (worm and worm wheel),

$$d_1 = m_x(q + 2x), \quad d_2 = m_x z_2. \quad (\text{AEV-T.2.4.1.3})$$

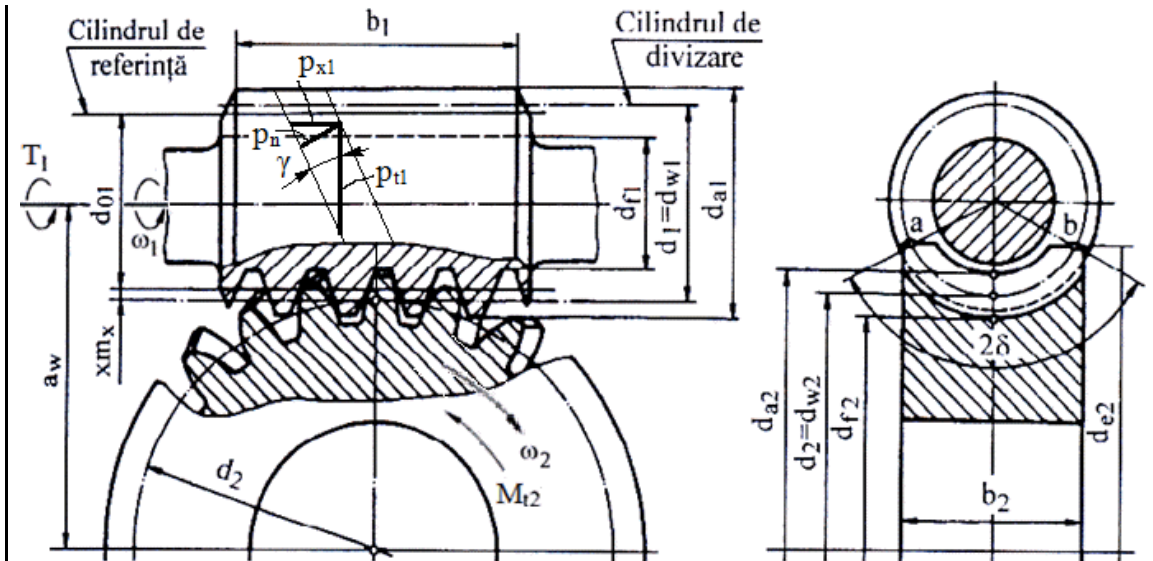
where x is the coefficient of the worm profile displacement.

Note: The globoidal worm gear, compared to the cylindrical worm gear, is characterized by quieter operation and increased bearing capacity, but has a lower efficiency and presents technological difficulties in the manufacture of the worm and assembly; it is used in the construction of small-sized transmissions (e.g. in steering boxes of motor vehicles)

Fig. AEV-T.2.4.1.2 Main geometric parameters of the cylindrical worm gear



a *b*
Fig. AEV-T.2.4.1.1 Cylindrical helical gear: *a* – spatial view; *b* – axial view



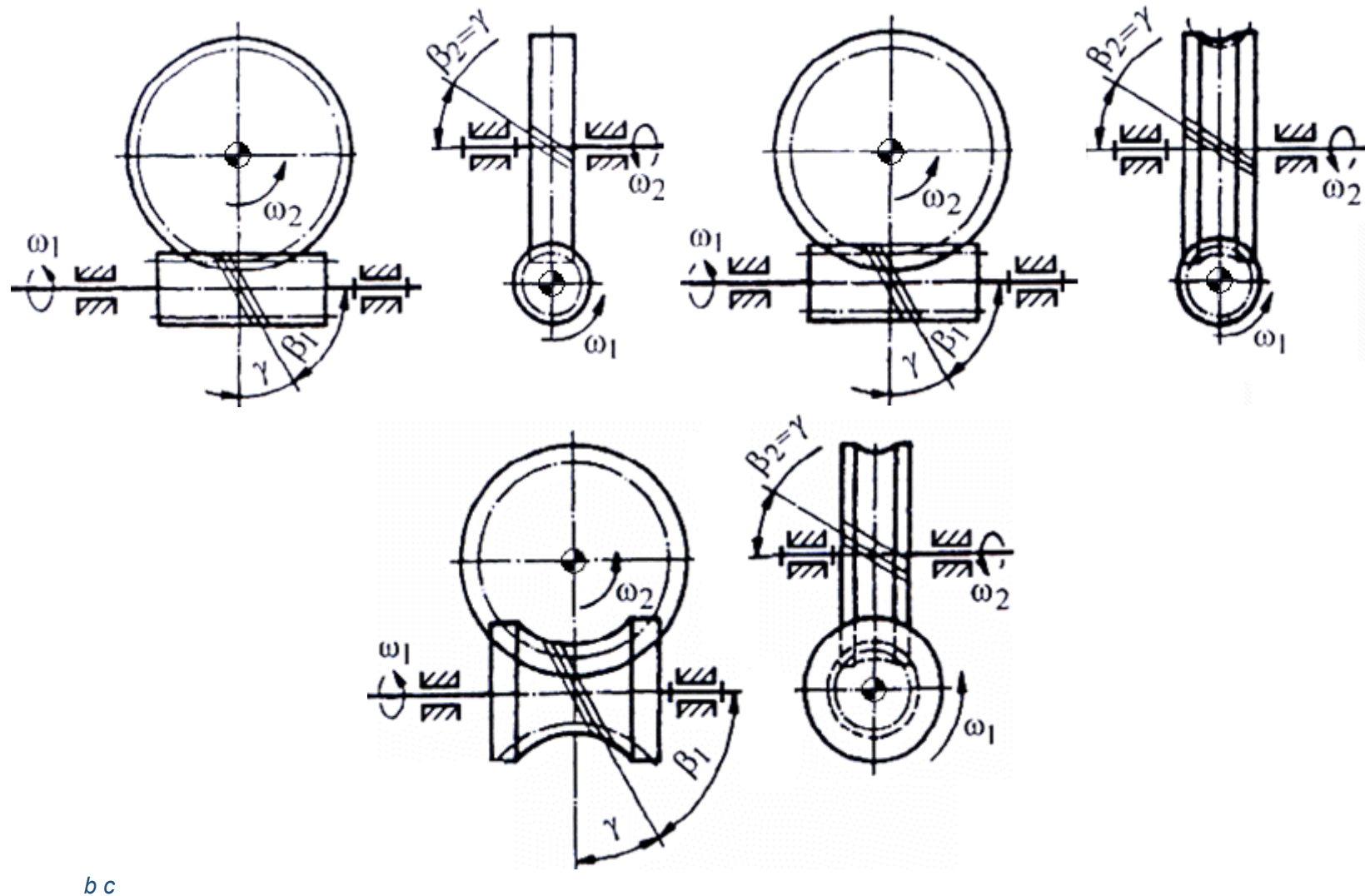
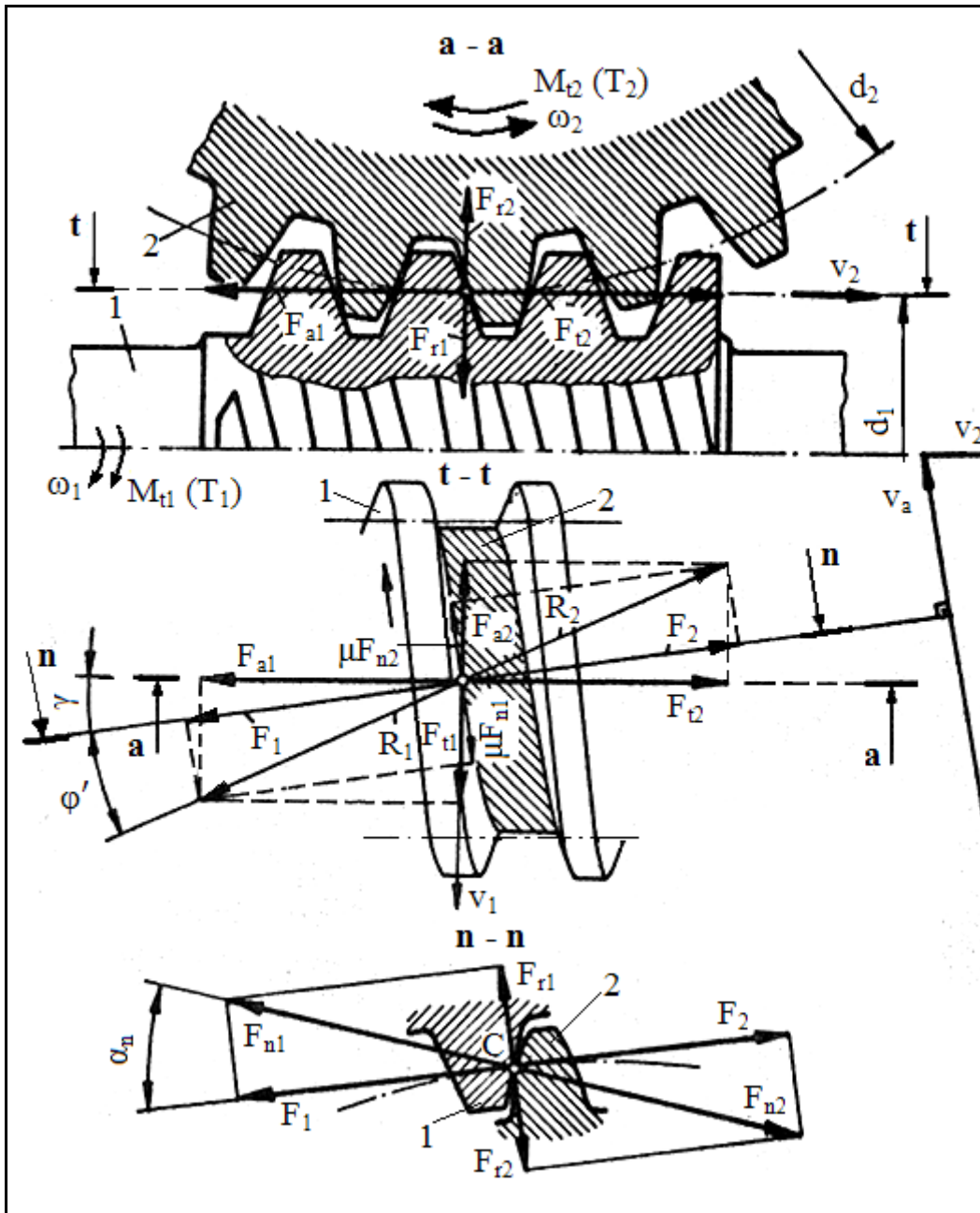


Fig. AEV-T.2.4.1.3 Gears with orthogonal crossed axes: a – cylindrical helical; b – cylindrical worm; b – globoidal worm

AEV-T.2.4.2 LOAD MODELING [Rădulescu, 1985]

Simplifying assumptions:

- normal forces are considered applied to the gear pole,
- forces are considered to be applied statically.



Tab. AEV-T.2.4.2.1 Force calculation relationships

Parameter and calculation relationship

Direction and meaning

Tangential and axial forces,

$$F_{t1} = F_{a2} = \frac{2 T_1}{d_1}, \quad (\text{AEV-T.2.4.2.1})$$

$$F_{t2} = F_{a1} = \frac{2 T_2}{d_2}, \quad (\text{AEV-T.2.4.1.2})$$

$$F_{t1} = F_{t2} \operatorname{tg}(\gamma + \phi') \quad (\text{AEV-T.2.4.1.3})$$

Normal force, $F_n = F_{n1} = F_{n2}$,

$$F_{n1} = \frac{F_{r1}}{\sin \alpha_n} = F_{t1} \frac{\cos \phi'}{\sin(\gamma + \phi') \cos \alpha_n} = T_1 \frac{2 \cos \phi'}{d_1 \sin(\gamma + \phi') \cos \alpha_n} \quad (\text{AEV-T.2.4.1.4})$$

$$F_{n2} = \frac{F_{r2}}{\sin \alpha_n} = F_{t2} \frac{\cos \phi'}{\cos(\gamma + \phi') \cos \alpha_n} = T_2 \frac{2 \cos \phi'}{d_2 \cos(\gamma + \phi') \cos \alpha_n} \quad (\text{AEV-T.2.4.1.5})$$

Direction, tangent to the pitch circle; $F_{t1,2}$, same direction as the speed of the worm, respectively, the worm wheel; $F_{a1,2}$, opposite direction to the wheel speed, respectively, opposite direction to the speed of the worm

Direction along the common normal of the profiles in contact

Fig. AEV-T.2.4.2.1 Worm gear force diagram (a-a axial section; n-n normal section; t-t tangential section)

Radial force, $F_r = F_{r1} = F_{r2}$,

$$F_{r1} = F_1 \operatorname{tg} \alpha_n = R_1 \cos \varphi' \operatorname{tg} \alpha_n = F_{t1} \frac{\cos \varphi' \operatorname{tg} \alpha_n}{\sin (\gamma + \varphi')} \quad (\text{AEV-T.2.4.1.6})$$

$$F_{r2} = F_2 \operatorname{tg} \alpha_n = R_2 \cos \varphi' \operatorname{tg} \alpha_n = F_{t2} \frac{\cos \varphi' \operatorname{tg} \alpha_n}{\cos (\gamma + \varphi')} \quad (\text{AEV-T.2.4.1.7})$$

Radial direction; direction towards the axis of the worm, respectively the wheel

Friction force,

$$F_f = \mu F_n, \quad (\text{AEV-T.2.4.1.8})$$

Reduced friction angle,

$$\varphi' = \operatorname{arctg} \frac{\mu F_n}{F_{1,2}} = \operatorname{arctg} \frac{\mu F_n}{F_{n1,2} \cos \alpha_n} = \operatorname{arctg} \frac{\mu}{\cos \alpha_n} \quad (\text{AEV-T.2.4.1.9})$$

The friction force has the same direction as the relative velocity, v_a , and the opposite direction to it.

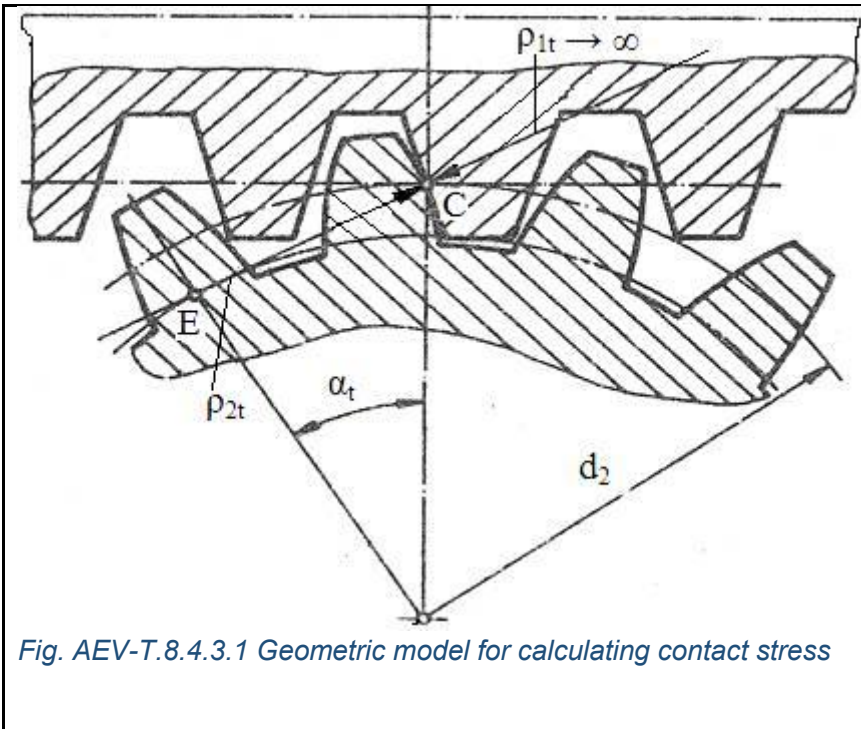
Note. $\eta = \frac{P_2}{P_1} = \frac{M_{t2} \omega_2}{M_{t1} \omega_1} = \frac{F_{t2} d_2 \omega_2}{F_{t1} d_1 \omega_1} = \frac{F_{t2} v_2}{F_{t1} v_1} = \frac{\operatorname{tg} \gamma}{\operatorname{tg}(\gamma + \varphi')}$ ($\eta = 0.65 \dots 0.8$, for $z_1 = 1$; $\eta = 0.8 \dots 0.92$, for $z_1 = 2$).
a. Gear efficiency,
b. In the simplified case, considering the friction negligible ($\mu = 0$, $\varphi' = 0$), the above force calculation relations become:

$$F_{t2} = F_{t1} \operatorname{tg} \gamma; \quad F_{r1} = F_{t1} \frac{\operatorname{tg} \alpha_n}{\sin \gamma}; \quad F_{r2} = F_{t2} \frac{\operatorname{tg} \alpha_n}{\cos \gamma}; \quad F_n = T_1 \frac{2}{d_1 \sin \gamma \cos \alpha_n} \quad (\text{tangential and axial forces are determined with rel. (AEV-T.2.4.1.1) and (AEV-T.2.4.1.2)}).$$

c. Meanings of the geometric parameters in fig. AEV-T.2.4.2.1: γ – lead angle of the worm; $d_{1,2}$ – diameters of the dividing circles of the worm and the worm wheel; α_n – angle of engagement in the normal plane; φ' – angle of friction; $\omega_{1,2}$ – angular velocity of the worm and the worm wheel; $M_{t1,2}$ or $T_{1,2}$ – torque of the worm and the worm wheel; $v_{1,2}$ – peripheral speeds of the worm and the worm wheel (in the gearing pole)

AEV-T.2.4.3 CONTACT STRESS CALCULATION MODEL

Tab. AEV-T.2.4.3.1 Definition and parameters of the contact calculation model [Jula, 1989; Rădulescu, 1985; Moldovean, 2001]]

 Fig. AEV-T.8.4.3.1 Geometric model for calculating contact stress	The calculation of the contact stress is done by analogy with that of cylindrical gears under the same calculation assumptions using <i>the Hertz model</i>, according to rel. (AEV-T.2.1.2.1) in subchapter. AEV-T.2.1.2 in which, the normal calculation force (considering the force F_n from tab. AEV-T.2.4.2.1, without friction), $F_{nc} = F_n K_A K_v K_{H\beta} K_\varepsilon \cong \frac{F_{t2}}{\cos \gamma \cos \alpha_n} K_A K_v K_{H\beta} K_{H\alpha} \quad (\text{AEV-T.2.4.3.1})$ ($K_{H\alpha} = 1$; $\gamma \cong \gamma_b$; unshifted gear), the length of the contact line of a worm wheel tooth (fig. AEV-T.2.4.1.2), $l_k = \frac{\widehat{ab}}{\cos \gamma} K_\varepsilon = \frac{\pi d_1}{\cos \gamma} \frac{2\delta}{360^\circ} 0,75 \varepsilon \cong 1,2 \frac{d_1}{\cos \gamma} \quad (\text{AEV-T.2.4.3.2})$ ($2\delta \cong 100^\circ$ (fig. AEV-T.2.4.1.2), $\varepsilon = 1.8 \dots 2.1$, contact ratio) and reduced curvature (fig. AEV-T.2.4.3.1), $\frac{1}{\rho} = \frac{1}{\rho_{2n}} = \frac{\cos \gamma}{\rho_{2t}} = \frac{\cos \gamma}{d_2 \sin \alpha} \quad (\text{AEV-T.2.4.3.3})$
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The contact calculation is done by means of the spur gear. The contact calculation relations are obtained by replacing the gear calculation parameters (tab. AEV-2.4.3.1) in the Hertz calculation relation of the contact stress ([AEV-T.2.1.2](#)). After performing the calculations, taking into account also the calculation relations of the geometric parameters (subchapter AEV-T.2.4.1), the maximum contact stress relations of the worm wheel are obtained from the following table.

Tab. AEV-T.2.4.3.2 Calculation relations for contact of cylindrical worm gear

Contact stress	sizing	Verification
$\sigma_{H2} = 1,24 Z_E Z_H \sqrt{\frac{2T_2}{d_2^3} \frac{z_2}{q} K_A K_v K_{H\beta}}$ (AEV-T.2.4.3.4)	$d_2 = 1,16 \sqrt[3]{T_2 \frac{z_2}{q} K_A K_v K_{H\beta} \left(\frac{Z_E Z_H}{\sigma_{HP2}}\right)^2}$ (AEV-T.2.4.3.5)	$\sigma_{H2} \leq \sigma_{HP2},$ (AEV-T.2.4.3.10) $\sigma_{HP2} = \frac{\sigma_H \lim Z_{N2}}{S_{H \min}} Z_L Z_R Z_v Z_w Z_\beta$
$\sigma_{H2} = 1,24 Z_E Z_H \sqrt{\frac{2T_2}{m_x^3} \frac{1}{z_2^2 q} K_A K_v K_{H\beta}}$ (AEV-T.2.4.3.6)	$m_x = 1,16 \sqrt[3]{\frac{T_2}{z_2^2 q} K_A K_v K_{H\beta} \left(\frac{Z_E Z_H}{\sigma_{HP2}}\right)^2}$ (AEV-T.2.4.3.7)	σ_{HP2} – <u>permissible contact stresses for pinion and wheel</u> ;

$\sigma_{H2} = 0,45 Z_E Z_H \sqrt{T_2 \left(\frac{1+q}{a} \right)^3 \frac{z_2}{q} K_A K_V I} \quad (\text{AEV-T.2.4.3.8})$	$a = 0,6 \left(1 + \frac{q}{z_2} \right)^3 \sqrt{T_2 \frac{z_2}{q} K_A K_V K_{H\beta} \left(\frac{Z_E Z}{\sigma_{HF}} \right)} \quad (\text{AEV-T.2.4.3.9})$	S_{Hmin} – <u>minimum safety factor for contact</u> (fer. AEV-M.1); Z_{N2} – <u>factors of the number of load cycles of the pinion and the wheel</u> (fer. AEV-F.13); Z_R – <u>flank roughness factor</u> (fer. AEV-F.11); Z_L – <u>lubricant factor</u> (fer. AEV-F.12); Z_v – <u>speed factor</u> (fer. AEV-F.10); Z_w – <u>hardness factor</u> (fer. AEV-F.9); Z_x – <u>size factor</u> (fer. AEV-F.8);
Obs. $Z_H = \sqrt{\frac{2 \cos \gamma}{\cos \alpha \sin \alpha}}$, <u>contact area factor</u> (fer. AEV-F.7)		

AEV-T.2.4.4 BENDING STRESS CALCULATION MODEL

The bending calculation is also done by means of the helical gear. The worm wheel is considered a helical cylindrical gear. The bending calculation relations are obtained by replacing the worm gear parameters in the calculation relations of the maximum bending stress of the wheel of a helical cylindrical gear from tab. [AEV-T.2.2.4.2](#). After performing the calculations, considering $Y_{sa2} = 1$, $K_{Fa} = 1$ and also taking into account the calculation relations of the geometric parameters (subchapter AEV-T.2.4.1), the calculation relation of the maximum contact stress of the worm wheel is obtained from the following table.

Tab. AEV-T.2.4.4.1 Calculation relations for bending of the cylindrical worm gear

Maximum bending stress (2- worm wheel)	Verification
$\sigma_{F2} = 1,7 \frac{T_2 z_1 z_2 K_A K_V K_{F\beta} Y_\beta}{q d_2^3 \cos \gamma} Y_{Fa2} \quad (\text{AEV-T.2.4.4.1})$	$\sigma_{F2} \leq \sigma_{FP2}$, (AEV-T.2.4.4.2) $\sigma_{FP2} = \frac{\sigma_{F \lim 2} Y_{N2} Y_{ST}}{S_{F \min}} Y_{\delta 2} Y_{R2} Y_x$ σ_{FP2} – allowable bending stresses for pinion and wheel; $\sigma_{F \lim 2}$ – <u>limit bending stress for pinion and wheel</u> (fer. AEV-M.1); $S_{F \min}$ – <u>minimum safety factor for bending</u> (fer. AEV-F.1); Y_{N2} – <u>factors of the number of load cycles of the pinion and the wheel</u> (fer. AEV-F.13); Y_{ST} – <u>reference bending stress factor</u> ($Y_{ST} = 2$); $Y_{\delta 2}$ – <u>stress concentrator factors of the pinion and wheel</u> (fer. AEV-F.15); Y_{R2} – <u>roughness factors of the pinion and wheel fillets</u> (fer. AEV-F.11); Y_x – <u>the size factor for bending</u> (fer. AEV-F.8).

AEV-T.2.4.5 THERMAL CALCULATION MODEL

The thermal equilibrium equation, $(1 - \eta)P_1 10^3 = k_t S (1 + \psi) (t_u - t_o)$ (AEV-T.2.4.5.1) where, P_1 [kW] is the transmitted power; k_t [W/m ² degree] – heat transfer factor (8...10, for weak air circulation; 14...16, for intense air circulation; 20...26, for fan cooling; 93...210, for oil cooling using a coil); S [m ²] – housing cooling surface (plus 50% of the surface of the ribs, if any); ψ – heat transfer coefficient through the housing bearing surfaces (usually, $\psi < 0.3$); t_u [°C] – oil temperature; t_o [°C] – ambient temperature.	Verification	sizing
	<u>Oil temperature,</u> $t_u = \frac{(1 - \eta)P_1 10^3}{k_t S (1 + \psi)} + t_o \leq t_{u\max} \quad (\text{AEV-T.2.4.5.2})$ where, $t_{u\max} = 60^\circ \dots 70^\circ \text{C}$, permissible oil temperature	<u>Required housing surface area,</u> $S = \frac{(1 - \eta)P_1 10^3}{k_t (1 + \psi) (t_{u\max} - t_o)} \quad (\text{AEV-T.2.4.5.3})$ <u>The required transfer factor,</u> $k_t = \frac{(1 - \eta)P_1 10^3}{S (1 + \psi) (t_{u\max} - t_o)} \quad (\text{AEV-T.2.4.5.4})$

ARD. SHAFTS AND AXLES

ARD-T. CONSTRUCTION AND MODELING OF SHAFTS AND AXLES

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ARD-T.1 DEFINITION

Straight shafts are machine elements with rotational movement intended to support other machine elements (gears, belt wheels, semi-couplings, flywheels, etc.) and transmit *torques*, which, in turn, are supported by bearings (sliding or rolling) on structures, usually fixed (chassis, frames, housings, etc.).

Axles are rotating or fixed machine parts, designed to support other rotating machine parts (they do not transmit torsional moments).

ARD-T.2 CONSTRUCTION STRUCTURE

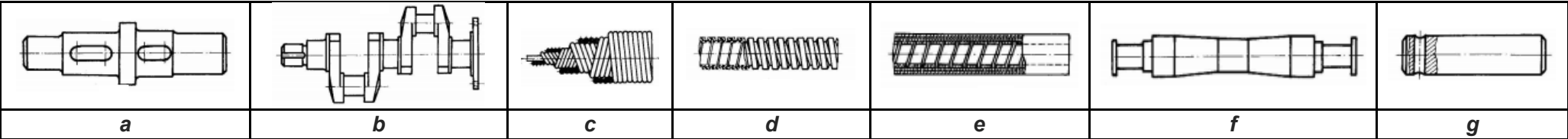


Fig. ARD-T.3.1 Types of shafts and axles: **a** – straight shafts; **b** - crankshafts; **c, d, e** – flexible shafts; **f** - rotating axle (for railway wagon); **g** - fixed axle (for pulley)

Note.

a. Crankshafts have two or more bearing journals, arranged along the length of the shaft, to ensure high structural rigidity, and one or more crankpin journals, for the connection with the connecting rods (of the engine mechanism); crankshafts can be provided with counterweights, for static and dynamic balancing.

b. Flexible shafts have a geometric axis with shapes that vary over time; they are made of layers of wire, tightly wound and in different directions, the winding direction of the last layer being opposite to the direction of rotation of the shaft, in order to achieve, during the transmission of movement, the tightening of the inner layers by the outer layer (fig. ARD-T.3.1, c); to protect the shaft against damage and dirt and to retain the grease between the turns, the flexible shaft is inserted into a metal sheath (fig. ARD-T.3.1, d) or made of rubberized fabric (fig. ARD-T.3.1, e); the flexible shaft is connected to the elements between which the movement is transmitted using end fittings.

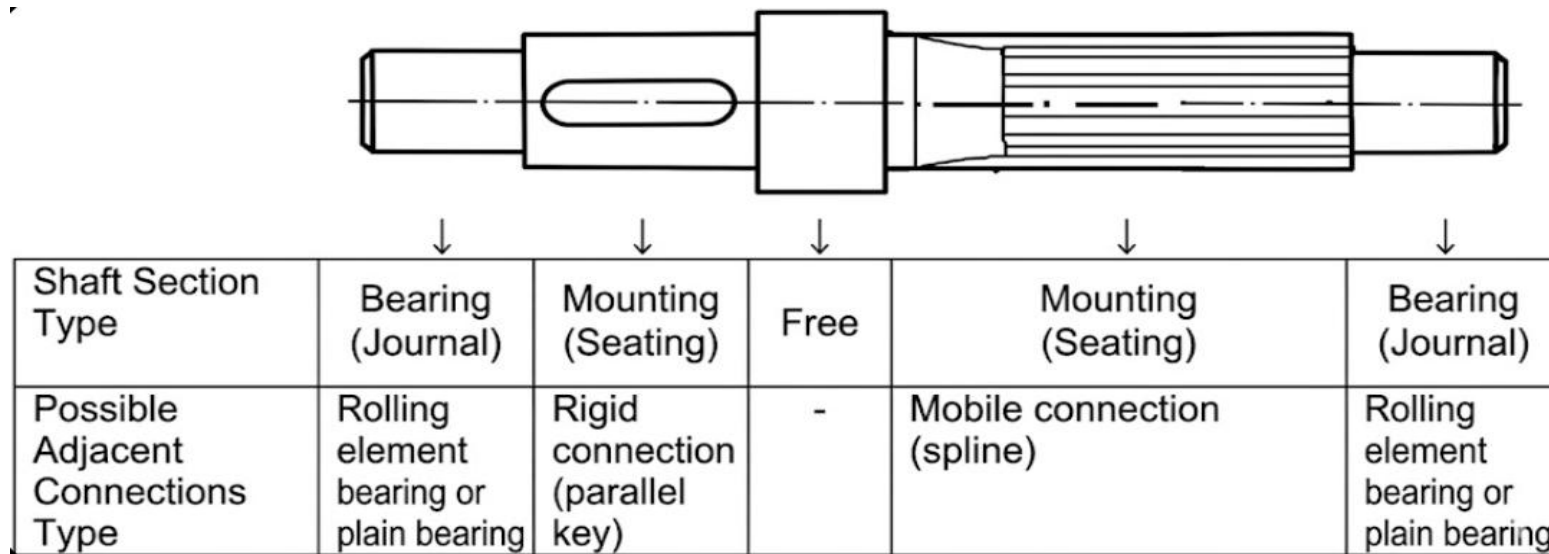


Fig. ARD-T.2.1 General construction structure of straight shafts

ARD-T.3 AREAS OF USE

Shafts are used in the transmissions of automobiles, tractors, agricultural machinery, technological equipment, machine tools, all gear transmissions, speed reducers, etc.

Crankshafts are used in the construction of connecting rod-crank mechanisms, to transform translational motion into rotational motion (in internal combustion engines) or vice versa (in compressors, presses, forging machines).

Flexible shafts are used to transmit torques between subassemblies that change their relative position during operation.

ARD-T.4 CLASSIFICATION

Tab. ARD-T.4.1 Classification of straight shafts [Moldovean, 1998]

Criterion	Shaft type	
Variation of shaft cross-section along length	With constant section (smooth) (fig. ARD-T.4.1, a)	With variable section, in steps (fig. ARD-T.4.1,b)

<i>Cross-sectional shape</i>	With solid section (fig. ARD-T.4.1, a,b,c)	Tubular (fig. ARD-T.4.1,d)	
<i>Rigidity (operating mode)</i>	Rigid	Elastic	
<i>Number of supports</i>	With two supports (statically determinate)	With three or more supports (statically indeterminate)	
<i>Position of the geometric axis</i>	Horizontal	Inclined	Vertical

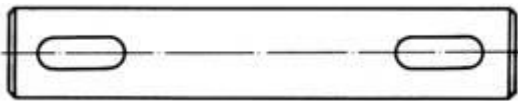
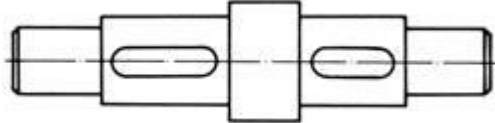
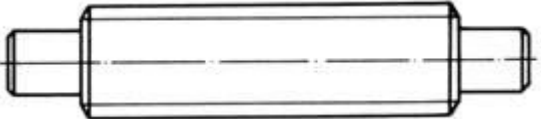
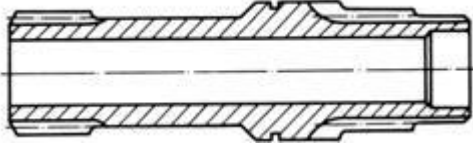
	
a	b
	
c	d

Fig. ARD-T.4.1 Main shapes of straight shafts: a - solid section; b - stepped; c - splined; d - tubular

ARD-T.5 MATERIALS AND HEAT TREATMENTS

For the manufacture of straight shafts, the following steels are frequently used:

- general-purpose structural steels (E295, E335, E360), which do not require heat treatment, for low loads;
- carbon steels for quenching and tempering (C 45, C55, etc.) and alloy steels for quenching and tempering (40 Cr 10, 41 CrNi 12, etc.), for medium loads and operating times;
- carbon case-hardening steels (C 15, C 20, etc.) and alloy case-hardening steels (13 CrNi 30), for heavy loads and high speeds.

Cast irons are used for the manufacture of large shafts because they are less sensitive to the effect of stress concentrators and dampen vibrations better.

The thermal or thermochemical treatments applied depend on the material from which the shafts are made, and can be: quenching and tempering, or quenching and tempering followed by surface hardening of the journals, splines, clamping portions, etc.; carburizing followed by hardening of the journals, clamping portions and splines; nitriding, etc.

ARD-T.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. ARD-T.6.1 Forms and causes of removal from service or improper behavior [Moldovean, 1998]

Failure / damage type	Forms	Appearance	Cause	Manifestation	Avoidance
Failure mode	Plastic deformation	Shafts made of soft materials under heavy stress	Exceeding the yield strength of the material	Permanent modification of the shape of the geometric axis followed by damage to adjacent elements (bearings, gears; fig. ARD-T.6.1, a)	Verification calculation for flexural deformations; overload protection
	Overload failure	Hard material shafts under heavy stress	Exceeding the material's breaking limit	Shaft breakage (fig. ARD-T.6.1, b)	Resistance calculation, overload protection
	Fatigue fracture	Shafts loaded with periodically varying loads	Exceeding the fatigue strength	Shaft breakage (fig. ARD-T.6.1, c)	Fatigue calculation; constructive solutions to increase fatigue resistance
Improper behavior	Flexural and/or torsional elastic deformations	Highly stressed shafts	High elasticity and heavy loads	Increased bending and torsional deflections leading to improper operation of gears and bearings	Verification calculation for flexural/torsional deformations; constructive stiffening solutions
	Vibrations	High-speed shafts	Manufacturing and assembly deviations under variable loads	Vibrations with increased amplitudes (resonance) leading to improper operation of gears and bearings	Vibration calculation (critical speed); constructive stiffening solutions
	Thermal deformations	Long shafts working in high temperature environments	Temperature increase	Elongation of shaft sections, which leads to improper functioning, especially of the bearings	Thermal calculation, solutions for compensating (not preventing) thermal deformations

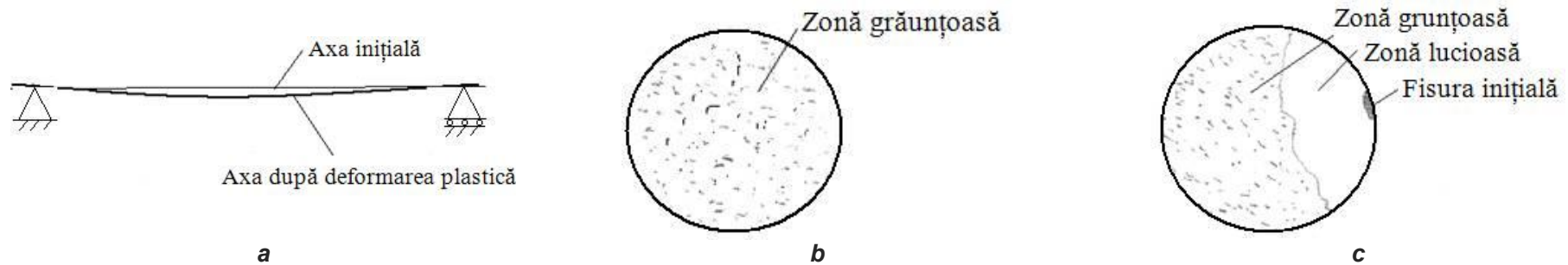


Fig. ARD-T.6.1 Forms of shaft removal from service: **a** – plastic deformation; **b** - overload failure; **c** - fatigue failure

ARD-T.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

Tab. ARD-T.7.1 Main functional and constructive parameters

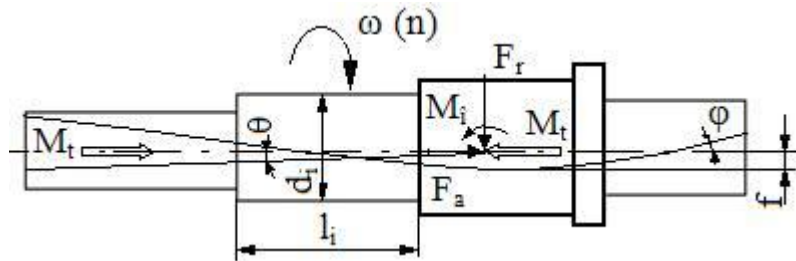


Fig. ARD-T.7.1 Scheme of main parameters

M_t [Nmm] – transmitted torque;
 ω [rad/s] (n [rot/min]) – angular velocity (speed), $\omega = \pi n/30$, $n = 30 \omega/\pi$;
 M_i [Nmm] – bending moment;
 F_r [N] – radial force;
 F_a [N] – axial force;
 f [mm] – bending deflection;
 φ [degrees] – bending rotation;
 θ [degrees] – torsional rotation (twisting);
 d_i, l_i [mm] – diameter, length of section i

ARD-T.8 CALCULATION MODELS OF STRAIGHT SHAFTS

ARD-T.8.1 CALCULATION MODELS OF SECTIONAL FORCES AND STRESSES

Straight shafts have 3D spatial structures that are externally loaded with distributed and time-varying loads.

Simplifying assumptions:

- shafts are considered to have one-dimensional structures;

- the distributed forces from the interaction areas of the shaft with the adjacent elements (supported and supporting) are considered equivalent to a system of concentrated forces according to equivalence schemes dependent on: the type of assembly, the rigidity of the mounted parts, etc. (fer. [ARD-P.1](#) - gears; fer. [ARD-P.2](#) - belts, chains);
- the schematization of shaft supports, by replacing them with conventional supports (with one or more degrees of freedom), takes into account the type of bearing (sliding, rolling), the rigidity of the shaft in the support area (fer. [ARD-P.3](#) - rolling bearings, fer. [ARD-P.4](#) - sliding bearings).

ARD-T.8.1.1 Calculation model of sectional forces and stresses of an intermediate shaft of a cylindrical speed reducer

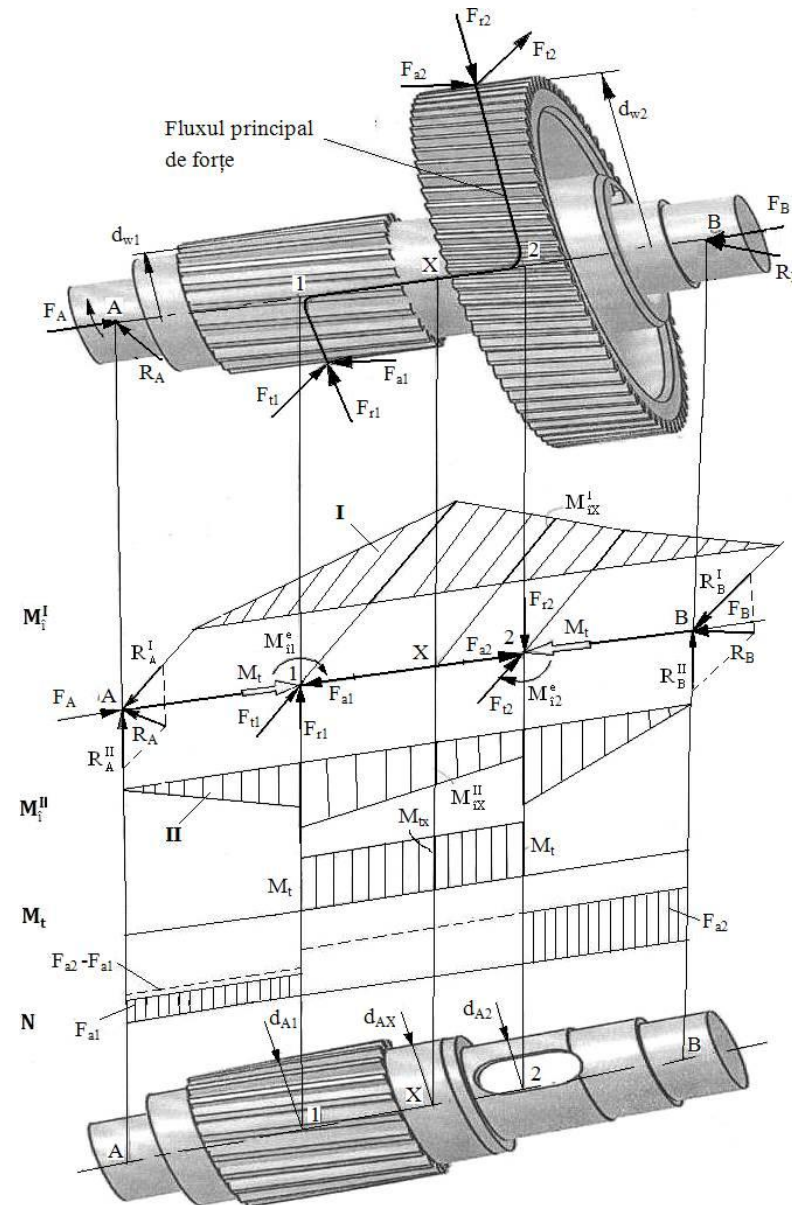


Fig. ARD-T.8.1 Calculation model of the forces and sectional stresses of the intermediate shaft

Tab. ARD-T.8.1.1 Calculation parameters and stress diagrams [Manea, 1970]**Geometric modeling**

The shaft is considered, based on the simplifying assumptions above, to be a straight bar, AB, with a stepped section (fig. ARD-T.8.1)

External loading

Pinion 1 (driver): normal force in the 2nd stage gear (see subchapter AEV-T.7.2.2, forces in the helical gear) with the components: *tangential force*, F_{t1} ; *radial force*, F_{r1} ; *axial force*, F_{a1} .

Wheel 2 (driven): normal force in the gear of stage I with the components: *tangential force*, F_{t2} ; *radial force*, F_{r2} ; *axial force*, F_{a2} .

Supports A and B

The bar associated with the shaft is supported in the radial bearing mounting area at points A and B where, according to the simplified support schemes in ARD-P.4, the reactions R_A and, respectively, R_B – *in the transverse plane* – and F_A and, respectively, F_B – *in the axial direction* are considered.

Shaft loading scheme with concentrated forces (at the axis level)

Section 1: tangential force, F_{t1} ; radial force, F_{r1} ; axial force, F_{a1} ; torsional and bending moments:

$$M_t = F_{t1} \frac{d_1}{2}, \quad M_{i1} = F_{a1} \frac{d_1}{2} \quad \text{, (ARD-T.8.1)}$$

Section 2: tangential force, F_{t2} ; radial force, F_{r2} ; axial force, F_{a2} ; torsional and bending moments:

$$M_t = F_{t2} \frac{d_2}{2}, \quad M_2 = F_{a2} \frac{d_2}{2} \quad \text{, (ARD-T.8.2)}$$

Sectional stress variation diagrams

The torsional moment, M_t , is constant on the section 1-2.

The bending moment, M_i , is spatially variable along the bar and, for the classical calculation, in correlation with the right-handed orthogonal coordinate system associated with the forces, is represented by the projections in the perpendicular planes, I and II. The calculation relation of the bending moment in section X is

$$M_{ix} = \sqrt{(M_{ix}^I)^2 + (M_{ix}^{II})^2} \quad \text{, (ARD-T.8.3)}$$

where, M_{ix}^I is the bending moment in plane I and, M_{ix}^{II} - the bending moment in plane II.

The axial sectional forces, N , resulting from the action of the axial forces in the gears, are constant along each shaft section, with values dependent on the direction of rotation (tab. ARD-T.8.1.2)

Tab. ARD-T.8.1.2 Axial sectional forces

Section	A-1	1-2	2-B	Axial reactions, $F_{A,B}$
Direction of rotation	Axial sectional force, N			
Counterclockwise	F_{a1}	-	F_{a2}	$F_A = F_{a1}; F_B = F_{a2}$
Clockwise	$F_{a2} - F_{a1}$	F_{a2}	-	$F_A = F_{a2} - F_{a1}; F_B = 0$
The shear forces have low values and are neglected.				

ARD-T.8.2 STRESS CALCULATION MODELS

ARD-T.8.2.1 Model for calculating stresses in the intermediate shaft of a cylindrical speed reducer

Under the action of the forces presented above, torsional, bending, compressive and shear (negligible) stresses appear in the shaft structure; they are unevenly distributed and, since the loads vary in time, they are also variable in time [Moldovean, 1998; Catrina, 2002].

Tab. ARD-T.8.1.3 Calculation relations for stresses in cross-section X (fig. ARD-T.8.1)

Stress type	Simple stress			Equivalent stress (according to the maximum shear stress theory)
	Torsion	Bending	Compression / tension	
Calculation relationships	$\tau_{tX} =$ $\frac{M_{tX}}{W_p} = \frac{M_{tX}}{\frac{\pi (d_A^X)^3}{16}} \quad (\text{ARD-T.8.4})$	$\sigma_{iX} = \frac{M_{iX}}{W_z} = \frac{M_{iX}}{\frac{\pi (d_A^X)^3}{32}} \quad (\text{ARD-T.8.5})$	$\sigma_{cX} = \frac{N_X}{A} = \frac{N_X}{\frac{\pi d_X^2}{4}} \quad (\text{ARD-T.8.6})$	$\sigma_{eX} = \sqrt{(\sigma_{iX} + \sigma_{cX})^2 + 4(\alpha \tau_{tX})^2} \quad (\text{ARD-T.8.7})$

Note.

a. The meanings of the parameters in the calculation relations: M_{tX} , M_{iX} and N_X are the torsional moment, the bending moment and, respectively, the axial force in section X (see rel. ARD-T.8.1, ARD-T.8.2, ARD-T.8.3 and tab. ARD-T.8.1.2); W_p , W_z and A – the polar section modulus, the axial section modulus and,

respectively, the cross-sectional area in section X ; – the diameter of the circular section X ; – the stress cycle factor. $d_A^X \alpha$

b. The bending and torsional stresses are not constant in time. Since the shaft is a rotating element (fig. ARD-T.8.2), the bending stress varies according to a symmetrical alternating cycle (fig. ARD-T.8.3,a). For the case of shaft rotation in one direction, the variation of the torsional stress, depending on the types of drive and the load to be overcome, can be equivalent to a pulsating cycle (fig. ARD-T.8.3,b) or a constant one (fig. ARD-T.8.3,c).

c. The cycles of variable loads are highlighted using the asymmetry factor R (fer. ARD-F.1), but, for design calculations, they are also marked with the symbols: I, for the constant cycle ($R = 1$); II, for the pulsating cycle ($R = 0$); III, for the symmetric alternating cycle ($R = -1$).

d. The factor from relation (ARD-T.8.7), which makes the component stresses equivalent in terms of their variation cycles (stresses varying according to cycles I and II with stresses varying according to cycle III), is determined with the relations: α

$\alpha = \sigma_{aIII} / \sigma_{aI}$, for the case of constant torque;

$\alpha = \sigma_{aiIII} / \sigma_{aiII}$, for the case of pulsating torque.

The indicative values of the stresses σ_{aiI} , σ_{aiII} , σ_{aiIII} corresponding to the three characteristic variation cycles, associated with common shaft materials, are presented in fer. [ARD-M.1](#).

e. In practical cases, for design calculations, the effect of axial forces is neglected (being much smaller than the other forces) and in relation (ARD-T.8.7) it is considered that $\sigma_{cX} = 0$.

The equivalent stress relation (ARD-T.8.7), neglecting the compressive stress ($\sigma_{cX} = 0$) and considering $W_p = 2W_z$, becomes:

$$\sigma_{eX} = \frac{\sqrt{(M_{iX})^2 + (\alpha M_{tX})^2}}{W_z} = \frac{M_{redX}}{W_z}, \quad (\text{ARD-T.8.8})$$

where M_{redX} is called the equivalent moment in section X.

Tab. ARD-T.8.1.4 Calculation parameters and relations

Calculation type	Verification	Sizing	Allowable stress
Calculation relationships	$\left(\frac{M_{redX}}{W_z}\right)_{X \in AB}^{\max} \leq \sigma_{aiIII} \quad (\text{ARD-T.8.9})$	$W_{znec}^X \geq \frac{M_{red}^X}{\sigma_{aiIII}} \quad (\text{ARD-T.8.10})$	$M_{red}^{cap} \geq \frac{W_z^{\min}}{\sigma_{aiIII}} \quad (\text{ARD-T.8.11})$
	$\frac{32}{\pi} \left(\frac{M_{redX}}{(d_A^X)^3}\right)_{X \in AB}^{\max} \leq \sigma_{aiIII} \quad (\text{ARD-T.8.12})$	$d_{Anec}^X \geq \sqrt[3]{\frac{32 M_{red}^X}{\pi \sigma_{aiIII}}} \quad (\text{ARD-T.8.13})$	$M_{red}^{cap} \geq \frac{\pi d_{min}^3}{32 \sigma_{aiIII}} \quad (\text{ARD-T.8.14})$

Note: Since the composite stress relation (ARD-T.8.8) depends nonlinearly on M_i , M_t and d_x , the verification of the shaft under composite stresses is performed in several cross-sections.

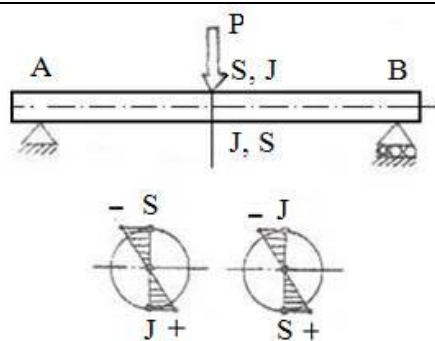


Fig. ARD-T.8.2 Schematic of the variation of bending stress in the transverse plane

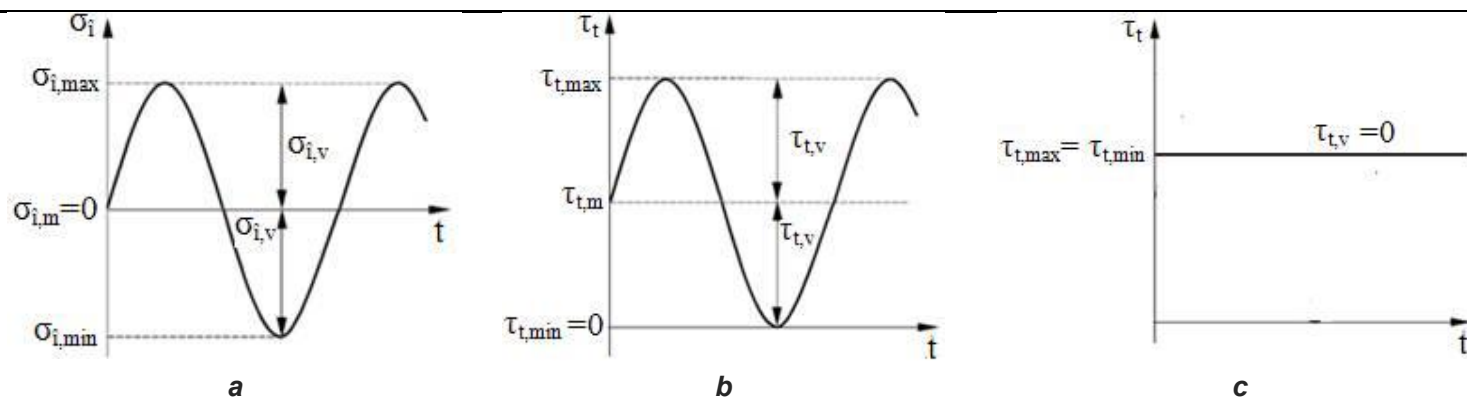


Fig. ARD-T.8.3 Stress variation cycles: **a** – symmetrical alternating (frequent for bending); **b** - pulsating (torsion); **c** – constant (torsion)

ARD-T.8.3 DEFORMATION (STIFFNESS) CALCULATION MODELS

Under the action of external loads, shafts are subjected to bending (flexural) and torsional deformations.

ARD-T.8.3.1 Bending deformation calculation model

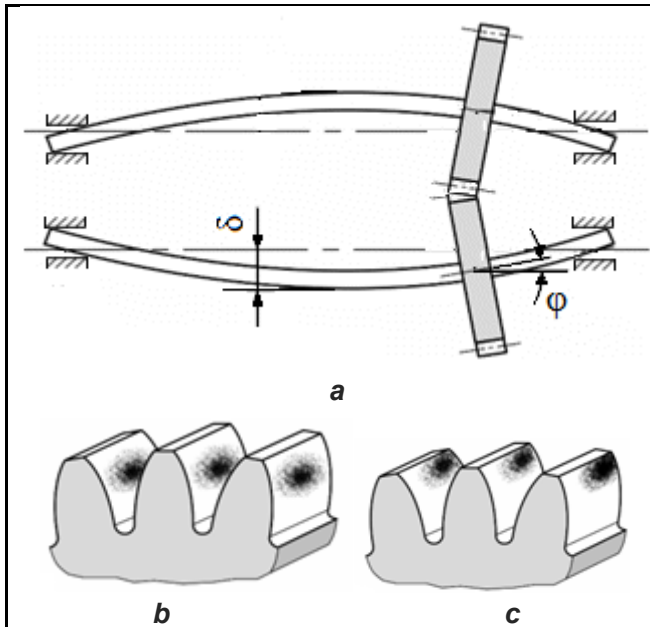


Fig. ARD-T.8.4 Real gear with elastic deformations: **a** – deformation scheme (deflections δ and slopes φ); **b** – contact spot in the case of rigid shafts (ideal); **c** – contact spot in the case of elastic shafts

Purpose, calculation features and limits

The calculation of flexural deformations, in general, is carried out in order to prevent improper operation (with transverse vibrations and noise) of supported elements (in particular, gears) and bearings (fig. ARD-T.8.4).

The calculation of bending deformations (for verification) consists of determining the values of deflections under the supported elements (gears) and angular deformations (rotations) in the bearings and limiting them to permissible values ($\delta \leq \delta_a$, $\varphi \leq \varphi_a$). The permissible values of bending deformations, recommended in the specialized literature, are: for deflections under the gears mounted on the shaft $\delta_a \leq (0.01 \dots 0.03)m$, m being the gear module, in mm; for angular deformations, in radians: $\varphi_a \leq 8 \cdot 10^{-3}$ – for radial ball bearings; $\varphi_a \leq 2.5 \cdot 10^{-3}$ – for radial cylindrical roller bearings; $\varphi_a \leq 1.7 \cdot 10^{-3}$ – for angular contact ball bearings or tapered roller bearings; $\varphi_a \leq 5 \cdot 10^{-2}$ – for self-aligning ball bearings or double-row cylindrical roller bearings; $\varphi_a \leq 10^{-3}$ – for sliding bearings.

The methods for calculating flexural deformations can be analytical, mainly studied in Strength of Materials, or numerically implemented in current high-performance packages. For simple cases, in the specialized literature, the equations of the elastic line and the expressions of the maximum deflections are given. Currently, the calculation of deformations is performed with high-performance software packages (e.g. MDESIGN) that take into account shafts with real shapes and dimensions. The classical (manual) calculation of bending deformations is based on the schemes used in the calculation of compound stresses (tab. ARD-T.8.1.3), the resulting deformation being obtained by the vector summation of the deformations in the two perpendicular loading planes of the shaft.

If the actual deformations exceed those recommended in the specialized literature (the verification conditions are not met), the bending rigidity of the shaft is increased by increasing its diameter.

ARD-T.8.3.2 Torsional deformation calculation model (twisting)

Purpose, calculation model and limits

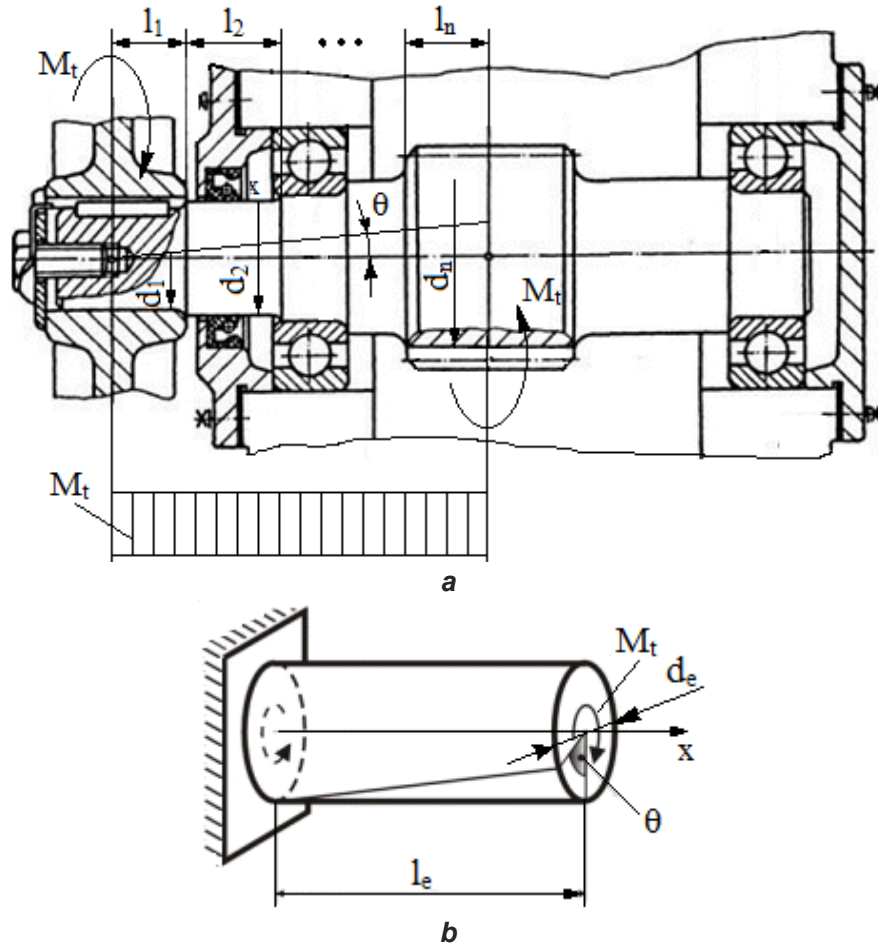


Fig. ARD-T.8.5 Calculation schemes for torsional deformations: **a** – stepped shaft; **b** – shaft with constant section (equivalent)

Purpose. The calculation of torsional (twisting) deformations, in general, is carried out in order to prevent improper operation (with torsional vibrations and noise) of the elements located along the motion and torque transmission path (fig. ARD-T.8.5).

Simplifying assumptions:

- the real model (fig. ARD-T.8.5,a) with the stepped shaft (with sections of diameter d_i and length l_i , $i = 1...n$) is replaced by an equivalent model (fig. ARD-T.8.5,b) with constant section (with diameter d_e) and length l_e ;
- the diameter d_e of the equivalent shaft is considered equal to one of the diameters of the sections of the real shaft (e.g. $d_e = d_j$ with $j = 1, 2$ or n);
- the length l_e of the equivalent shaft is determined from the conditions of equality of the stiffnesses of each section of the real shaft with that of the equivalent one,

$$\frac{G I_{pi}}{l_i} = \frac{G I_e}{l_{ie}}, \quad (\text{ARD-T.8.3.2.1})$$

from which, taking into account that the polar moments of inertia of the sections i and of the section corresponding to the equivalent shaft $I_{pi} = \pi d_i^4 / 32$ and, respectively, $I_e = \pi d_e^4 / 32$, the length of the equivalent section results,

$$l_{ie} = l_i \left(\frac{d_e}{d_i} \right)^4 \quad (\text{ARD-T.8.3.2.2})$$

The length of the equivalent shaft is determined as the sum of the lengths of the equivalent sections,

$$l_e = \sum_{i=1}^n l_{ie} = \sum_{i=1}^n l_i \left(\frac{d_e}{d_i} \right)^4 \quad (\text{ARD-T.8.3.2.3})$$

The twist angle (axial rotation) of the equivalent shaft and therefore of the real shaft in the area subjected to torsion is determined by the relationship,

$$\theta = \frac{M_t l_e}{G I_e} = \frac{32 M_t}{\pi G d_e^4} \sum_{i=1}^n l_i \left(\frac{d_e}{d_i} \right)^4 \quad [\text{rad}], \quad (\text{ARD-T.8.3.2.4})$$

where, $d_e = d_j$ [mm] with $j = 1, 2$ or n ; M_t [Nmm] - torsional moment; G [MPa] - transverse modulus of elasticity of the material; l_i ($i = 1...n$) [mm] - lengths of the real shaft sections

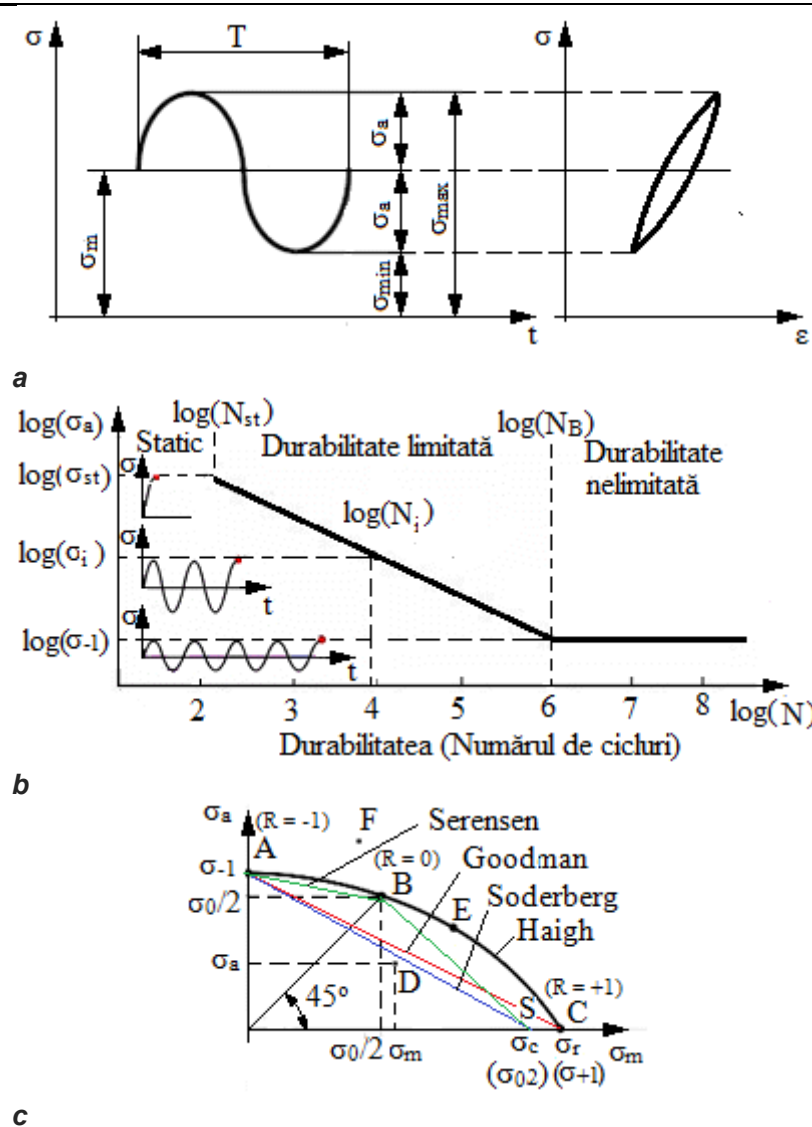
The verification calculation for torsional (twisting) deformations consists of determining the values of the torsion angles (axial rotations) of the shaft and limiting them to admissible values ($\theta \leq \theta_a$).

The permissible values for the torsion angle (twisting), determined by the purpose of the shaft, recommended in the specialized literature, are: $\theta_a \leq (15... 25) 10^{-2}$ rad/m, for motor vehicle differential shafts (planetary shafts); $\theta_a \leq (40... 55) 10^{-4}$ rad/m, for shafts of the travel mechanisms of overhead cranes. In the case of

short shafts (general-purpose reducers, gearboxes, etc.), in general, torsional rigidity does not play an essential role and, as a result, calculation for torsional deformations is not necessary.

If the effective torsion (twisting) angles exceed the values recommended in the specialized literature, the torsional rigidity of the shaft is increased by increasing the diameters and/or decreasing the lengths of its sections.

ARD-T.8.4 CALCULATION MODELS FOR VARIABLE LOADS (FATIGUE)



Purpose, calculation features and limits

Purpose. The calculation for variable stresses (fatigue), in general, is performed in order to prevent shaft breakage, especially in the area of stress concentrators (fig. ARD-T.6.1).

The usual calculation of shafts under variable loads consists of a verification under stationary (simple or compound) loads, as a rule, for unlimited durability (fig. ARD-T.8.6,b). This calculation is made starting from the durability diagram (of the material's limit cycles, fig. ARD-T.8.6,c) for which a simplified, schematic diagram is developed (Soderberg, Goodman, Serensen; fig. ARD-T.8.6,c), on the basis of which calculation relations are established for the safety factor, as the ratio between the fatigue resistance of the material (maximum stress of the limit cycle, fig. ARD-T.8.6,c; see Note, c) and the maximum stress of the variable load cycle (fig. ARD-T.8.6,a).

The fatigue resistance condition of straight shafts in areas where there are stress concentrators (keyways, grooves, diameter jumps, cross holes, threads, press fits, etc.; fig. ARD-T.8.7), especially when subjected to compound stresses (torsion and bending) is given by the relationship,

$$c \geq c_a, \text{ (ARD-T.8.4.1)}$$

which requires the calculation of the global safety factor in areas with stress concentrators (fig. ARD-T.8.7),

$$c = \frac{c_\sigma c_\tau}{\sqrt{c_\sigma^2 + c_\tau^2}}, \quad \text{(ARD-T.8.4.2)}$$

which is determined based on the partial safety coefficients (according to the simplified schematization, Serensen),

$$c_\sigma = \frac{\sigma_{-1}}{\frac{\beta_{K\sigma}}{\varepsilon_\sigma \gamma_\sigma} \sigma_v}, \quad c_\tau = \frac{\tau_{-1}}{\frac{\beta_{K\tau}}{\varepsilon_\tau \gamma_\tau} \tau_v + \frac{2\tau_{-1} - \tau_0}{\tau_0} \tau_m}, \quad \text{(ARD-T.8.4.3)}$$

where, σ_{-1} , τ_{-1} are the fatigue resistance for bending and torsional stress, respectively, for the symmetric alternating cycle; τ_0 – fatigue resistance for torsional stress, for the pulsating cycle; τ_m – average stress of the torsional stress cycle; σ_v , τ_v – amplitudes of the bending and torsional stress cycles, respectively; $\beta_{K\sigma}$, $\beta_{K\tau}$ – effective stress concentration coefficients in the

Fig. ARD-T.8.6 Fatigue calculation schemes for shafts: **a** – deterministic (stationary) load cycle scheme; **b** – durability curve, σ -N (Wöhler), in logarithmic coordinates; **c** – fatigue strength diagrams (limit cycles, for $R = [-1, 1]$)

considered section, corresponding to bending and torsional stress, respectively; ε_{σ} , ε_{τ} – dimensional coefficients, corresponding to bending and torsional stress, respectively; γ_{σ} , γ_{τ} – surface quality coefficients, corresponding to bending and torsional stress, respectively. For the admissible safety coefficient, the following values are recommended: $c_a = 1.3 \dots 1.5$ – for shafts made of homogeneous material, with correct execution technology and for which the stresses are precisely established; $c_a = 1.5 \dots 2.5$ – for shafts made of inhomogeneous material and for which the stresses are approximately established.

Note.

a. *Deterministic (stationary) stress cycles* described by periodic functions (frequently sinusoidal) are characterized by the following characteristic parameters (fig. ARD-T.8.6,a): maximum stress, $\sigma_{\max}$; minimum stress, $\sigma_{\min}$; average stress, $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2$; cycle amplitude, $\sigma_a = (\sigma_{\max} - \sigma_{\min})/2$; asymmetry coefficient, $R = \sigma_{\min}/\sigma_{\max}$.

b. The *functional states of a shaft from the point of view of fatigue resistance* are (fig. ARD-T.8.6,b): with low durability (reduced service life, $N < 10^2 \dots 10^3$); with limited (high) durability ($10^2 \dots 10^3 < N < 10^6 \dots 10^7$); with unlimited (high) durability ($N > 10^6 \dots 10^7$).

c. The *stress cycle with the asymmetry coefficient R and the maximum stress $\sigma_{\max}$ equal to the fatigue strength of the material (σ_{-1}) is called the limit cycle*. The effect of the average stress is quantified with the *fatigue strength diagram* or *the limit cycle curve* (the locus of the points corresponding to the limit cycles) which highlights the variation of the fatigue strength with the asymmetry coefficient, $-1 < R < 1$ (Haigh schematization, fig. ARD-T.8.6,c). In practice, these diagrams allow the determination of the fatigue strengths as a function of the asymmetry coefficients R (or of σ_m and σ_a). Fig. ARD-T.8.6,c presents the Haigh diagram for tensile stress, highlighting points corresponding to the stress cycles: A – symmetrical alternating, B – pulsating, C – static, E – alternating/oscillating ($-1 < R < 1$).

d. The coefficients $\beta_{k\sigma}$, $\beta_{k\tau}$, γ_{σ} , γ_{τ} , ε_{σ} , ε_{τ} are correction coefficients, which take into account the fact that the fatigue tests of the materials are performed on standard specimens that are performed without stress concentrators and whose dimensions and processing differ from those of the designed shafts (fer. ARD-P.1). The coefficients $\beta_{k\sigma}$, $\beta_{k\tau}$, are defined as the ratio between the fatigue resistance of the polished specimen, without a concentrator, and the fatigue resistance of the specimen with the same dimensions as the smooth one but provided with a concentrator, for the same stress cycle; their values are chosen from the diagrams, depending on the type and characteristics of the concentrator (fer. ARD-P.1). The coefficients ε_{σ} and ε_{τ} take into account the fact that the actual dimensions of the

shaft differ from those of the specimen, their values being given in the diagrams and are in the range 1.4... 19 (fer. ARD-P.1). The coefficients γ_σ and γ_τ are defined as the ratio between the fatigue resistance of the specimen with a rough surface and the fatigue resistance of the specimen with a polished surface (fer. ARD-P.1).

e. If in a certain section the condition imposed by the relation (ARD-T.8.4.1) is not met, *constructive measures* are taken to meet it by adopting concentrator shapes that induce much lower local stresses (fig. ARD-T.8.7): section transitions (diameter jumps, fig. ARD-T.8.7,a...l); keyways or splines (fig. ARD-T.8.7,m,n,o); reduction of edge pressures in the shaft – supported part contact areas (fig. ARD-T.8.7,p...t); threads – it is recommended to place threads at the shaft ends, where bending moments are reduced and/or use fine pitch threads.

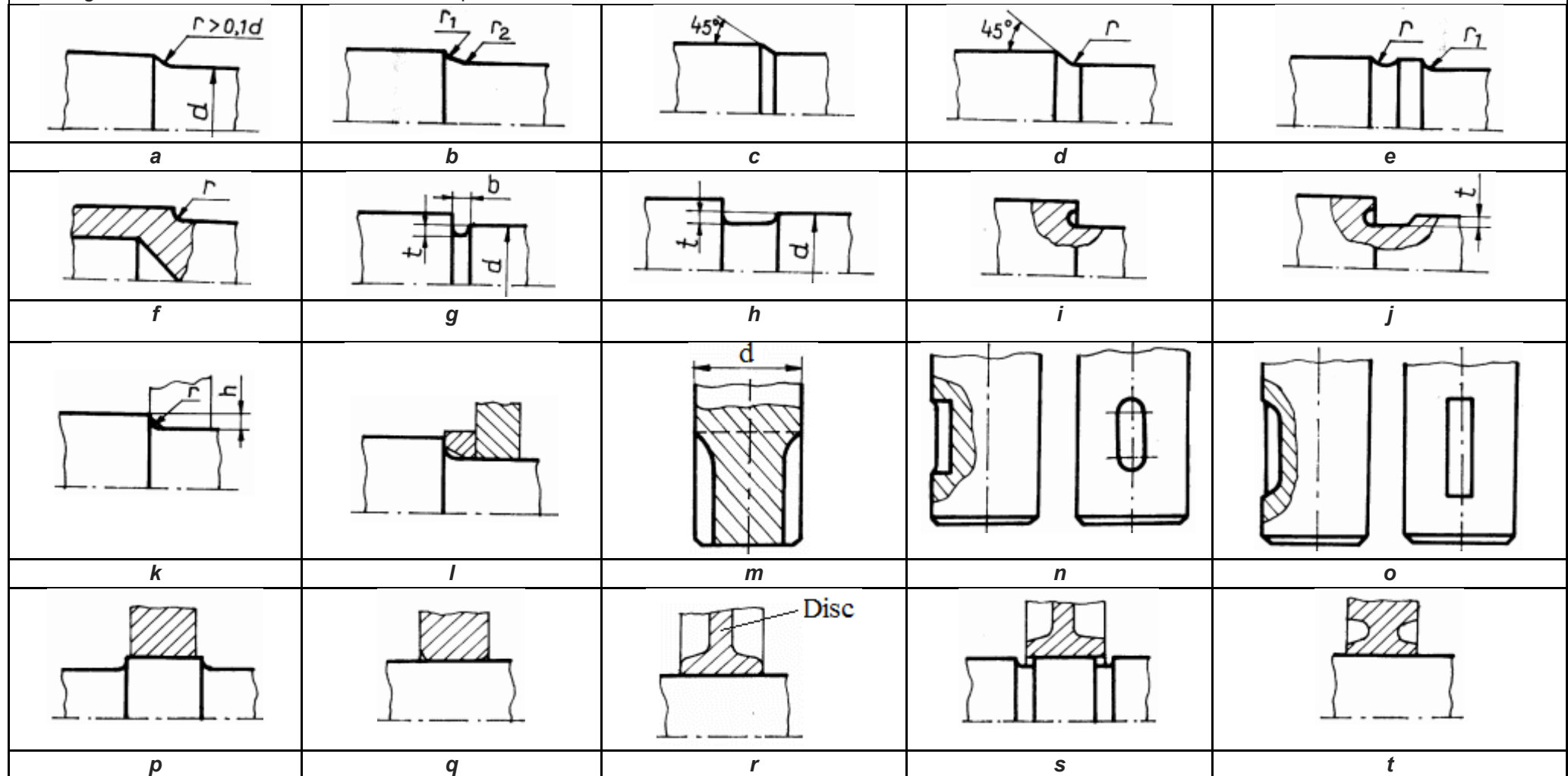


Fig. ARD-T.8.7 Construction measures to increase fatigue resistance: **a** – connection radius (in case the difference between the steps is small); **b** - two different connection radii (in case of highly stressed shafts); **c** - chamfering of the end of the large diameter step (for small diameter jumps); **d** - chamfering of the end of the large diameter step, combined with connection to the small diameter step (for large diameter jumps); **e** - connection radius, combined with a discharge channel on

*the large diameter step; **f** - connection radius, combined with the execution of a hole on the large diameter step; **g** - short passage channel, executed at the end of the small diameter step (for small shafts); **h** - long passage channel, executed at the end of the small diameter step (for large shafts); **i** - internal clearance, made in the large diameter step (in case of reduced deviations from perpendicularity); **j** - passage channel combined with internal clearance (the solution ensures increased fatigue resistance, access of the grinding stone along the entire length of the small diameter section and correct axial support of the components mounted on the shaft); **k** - large connection radius requiring chamfering of the conjugate part; **l** - use of intermediate parts (rings); **m** - grooves (higher fatigue resistance than in the case of keyways); **n** - keyways with rounded ends (since they do not provide increased fatigue resistance, they are recommended to be positioned towards the ends of the shafts); **o** - keyways with straight ends (increased fatigue resistance, they are positioned in the central areas of the shafts); **p** - thickening of the seating portion; **q** - chamfering or rounding of the edges of the hub; **r** - disk wheel configuration (for large diameter wheels); **s** - execution of discharge channels in the shaft; **t** - execution of discharge channels in the hub*

ARD-T.8.5 VIBRATION CALCULATION MODELS

The purpose of the vibration calculation is to prevent shaft breakage due to the greatly increased stresses that occur when the subassembly enters resonance. The calculation consists of determining the critical speed of the shaft and comparing it with the operating speed.

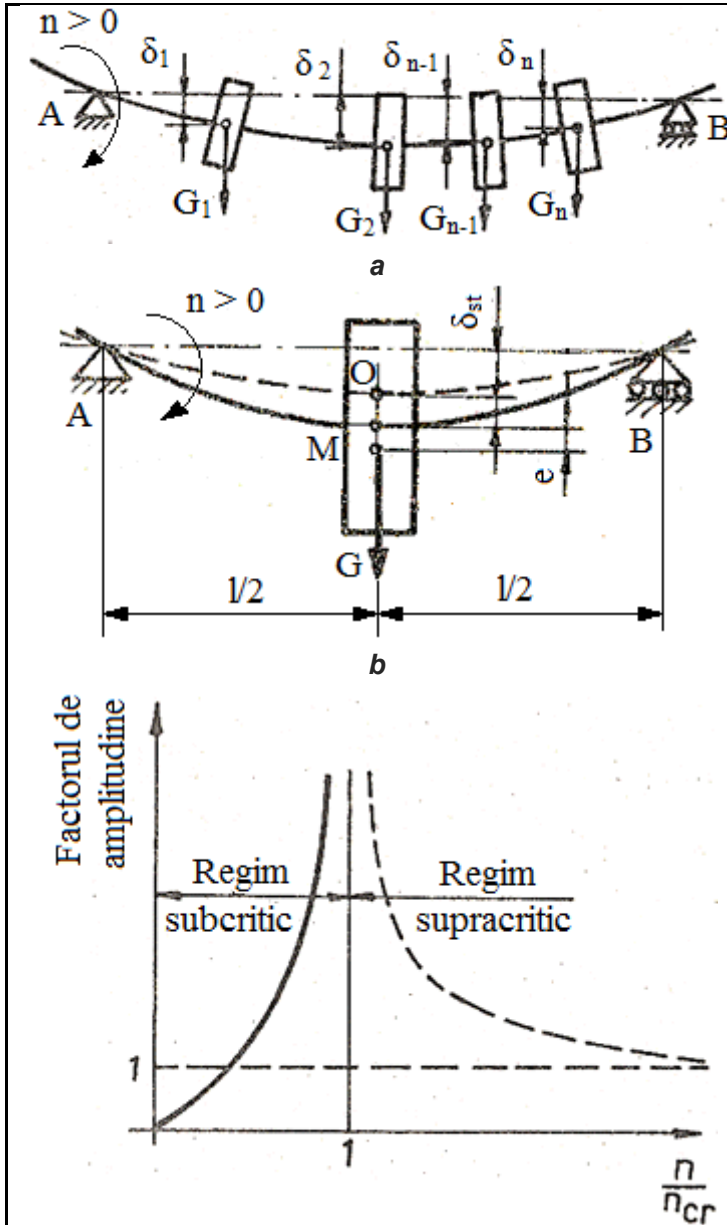
The critical speed is the speed at which the shaft enters resonance as a result of the situation in which the natural frequency (pulsation) coincides with or is a multiple of the frequency of the disturbing forces.

The natural frequency (pulsation) is a characteristic of an oscillating system independent of the initial conditions of the motion.

Depending on the type of disturbing forces, flexural (transverse, bending), torsional and/or longitudinal vibrations are encountered (in vertical shafts where variable axial loads occur).

ARD-T.8.5.1 Calculation model for transverse (flexural) vibrations

Flexural (transverse) vibrations are caused by errors in the execution and assembly of the shaft and the elements mounted on it, by the inhomogeneities of the material and by its elastic deformation under the action of external forces.



c

Fig. ARD-T.8.8 Calculation model for flexural vibrations: **a** – general case with n discs; **b** – particular case with a symmetrical disc; **c** – operating modes

Flexural vibration calculation model

Simplifying assumptions:

- the shaft subassembly is considered to be composed of a shaft with negligible mass and n discs (gear wheels, belt wheels, flywheels, etc.) mounted on the shaft that have masses m_i , $i = 1 \dots n$;
- due to execution and assembly errors and material inhomogeneities, the centers of mass of the discs are eccentric (fig. ARD-T.8.8, b) and during shaft rotation ($n > 0$) centrifugal (excitation) forces appear that lead to transverse oscillations;
- the elastic line (fig. ARD-T.8.8, a), at rest ($n = 0$), represents the extreme limit of the deformations that occur during rotation and is determined by the deflections δ_i produced by the weights, $G_i = m_i g$ (g , gravitational acceleration);
- considering harmonic vibrations, the transverse deflections vary in time according to the relationship, $\delta_{ti} = \delta_i \sin \omega t$;
- the velocities of the disc centers,

$$v_{ti}(t) = \frac{d\delta_{ti}(t)}{dt} = \omega \delta_i \cos \omega t$$

have the maximum values, $v_i = \omega \delta_i$.

Calculation relations:

From the condition of equalizing the maximum potential energy (corresponding to the extreme position of the elastic line where the kinetic energy is zero) with the maximum kinetic energy (corresponding to the mean position of the elastic line, line AB, where the potential energy is zero) of the shaft subassembly,

$$\sum_{i=1}^n \frac{G_i \delta_i^2}{2} = \sum_{i=1}^n \frac{m_i v_i^2}{2} \quad (\text{ARD-T.8.5.1})$$

considering the simplifying assumptions, the critical angular velocity [rad/s] (speed [rot/min]) results (at which resonance occurs),

$$\omega = \omega_{cr} = \sqrt{\frac{g \sum_{i=1}^n m_i \delta_i}{\sum_{i=1}^n m_i \delta_i^2}} \quad \text{sau} \quad n = n_{cr} = \frac{30}{\pi} \sqrt{\frac{g \sum_{i=1}^n m_i \delta_i}{\sum_{i=1}^n m_i \delta_i^2}} \quad (\text{ARD-T.8.5.2})$$

where, m_i [kg] are the masses of the discs, g [mm/s²] - gravitational acceleration, δ_i [mm] - shaft deflections in the disc areas.

For the particular case of a shaft with a single disc (fig. ARD-T.8.8, b) mounted symmetrically on the shaft, these relations become,

$$\omega = \omega_{cr} = \sqrt{\frac{g}{\delta}} \quad \text{sau} \quad n = n_{cr} = \frac{30}{\pi} \sqrt{\frac{g}{\delta}} \quad (\text{ARD-T.8.5.3})$$

Considering, for this case, in the above relationship,

$$\delta = \delta_{st} = \frac{G l^3}{48 E I_z}, \quad (\text{ARD-T.8.5.4})$$

the critical speed relation results,

$$n_{cr} = \frac{30}{\pi} \sqrt{\frac{48 E I_z}{m l^3}}, \quad (\text{ARD-T.8.5.5})$$

where, E [MPa] is the longitudinal modulus of elasticity of the shaft material; I_z [mm⁴] - axial moment of inertia of the shaft section; m [kg] - mass of the disc mounted on the shaft; l [mm] - shaft length

Operating modes:

a. $n/n_{cr} < 1$ ($n < n_{cr}$), the subcritical regime (fig. ARD-T.8.8,c), corresponding to rigid shafts;

b. $n/n_{cr} > 1$ ($n > n_{cr}$), the supercritical regime (fig. ARD-T.8.8,c) corresponding to elastic shafts (there is a tendency to self-centering); it is recommended to pass over the critical speed quickly and/or to adopt constructive measures to limit the amplitudes of vibrations at resonance.

Note: It is recommended to avoid the domain d in the resonance zone, $n/n_{cr} = 0.85 \dots 1.25$ in which the increase in deformations (deflections) is very high.

ARD-T.8.5.2 Calculation model for torsional vibrations

ARD-T.9 AXLE CALCULATION MODELS

Axle calculation features

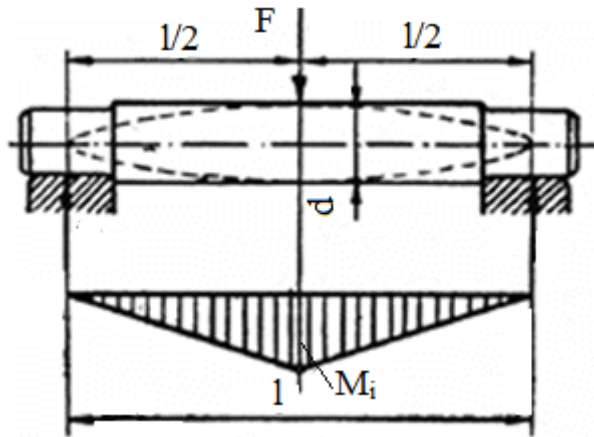


Fig. ARD-T.8.9 Calculation scheme of an axle

Under the action of external loads, axles, in most situations, are subjected to bending stresses.

From the condition of bending strength,

$$\sigma_i = \frac{M_i}{W_z} \leq \sigma_{a\hat{i}}, \quad (\text{ARD-T.8.9.1})$$

the dimensioning relationship is determined,

$$W_z = \frac{M_i}{\sigma_{a\hat{i}}}, \quad (\text{ARD-T.8.9.2})$$

from which the axle diameter in the median area is determined,

$$d = \sqrt[3]{\frac{32 M_i}{\pi \sigma_{a\hat{i}}}} \quad (\text{ARD-T.8.9.3})$$

RUL. ROLLING BEARINGS

RUL-T. CONSTRUCTION AND MODELING OF ROLLING BEARINGS

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RUL-T.1 DEFINITIONS

Rotational bearings are subassemblies whose main function is to support (carry) shafts, axles or other machine elements and subassemblies with rotational motion, being able to take up the loads (forces and/or moments) acting on them. *Rolling-contact bearings* include one or more independent specialized assemblies of the rolling bearing type.

The rolling bearing arrangement is the system consisting of two or more rolling-bearing supports whose purpose is the radial and axial location in both directions of the supported element (usually a shaft carrying gears or other elements), without inducing additional forces in the bearings under thermal effects and external and internal forces (elastic deformations).

RUL-T.2 CONSTRUCTION STRUCTURE

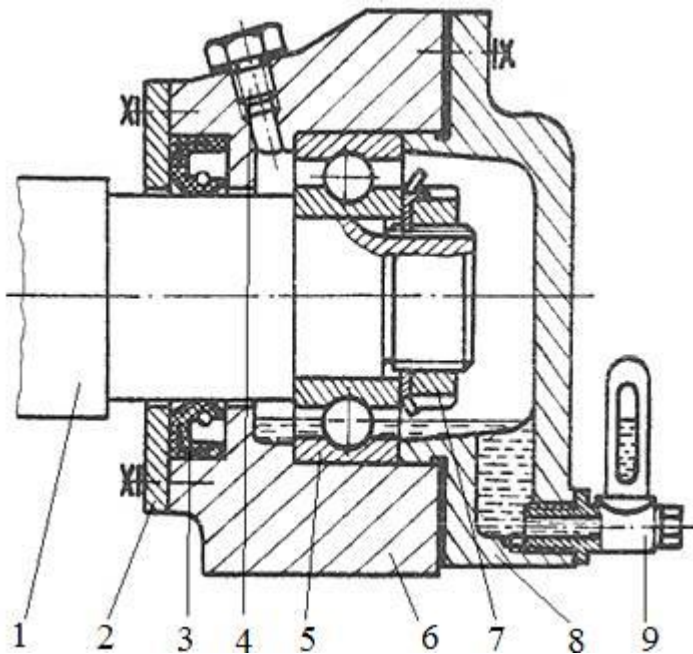


Fig. RUL-T.2.1 Construction structure of a bearing support with one rolling bearing

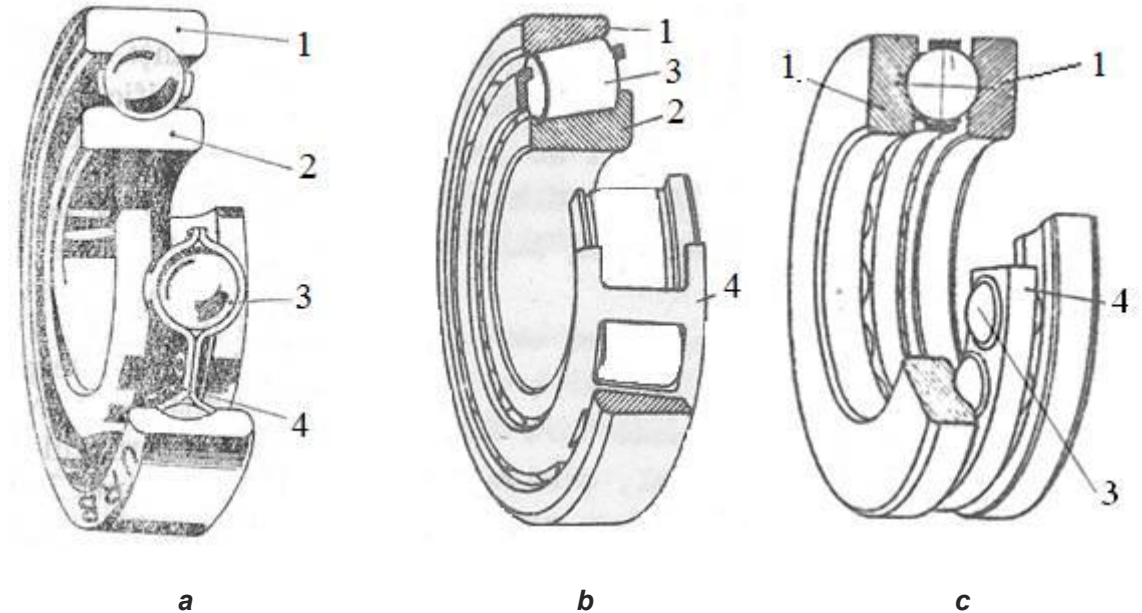


Fig. RUL-T.2.2 General structure of rolling bearings: **a** – deep groove (radial) ball bearing; **b** – tapered roller bearing (radial-axial); **c** – single-direction thrust ball bearing

Note.

- a. Meaning of the elements of the bearing support in fig. RUL-T.2.1:** 1 – shaft, 2 – cover, 3 – rotary lip seal, 4 – breather plug, 5 – deep groove ball bearing, 6 – housing, 7 – slotted nut with lock washer, 8 – cover, 9 – oil level indicator.
- b. Meaning of the bearing elements in fig. RUL-T.2.2:** 1 – outer ring, or housing washer for the single-direction thrust ball bearing, 2 – inner ring, 3 – rolling element; 4 – cage (independent element or subassembly)

RUL-T.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Transmissions of automobiles, tractors, agricultural machinery, technological equipment and machine tools, all gear transmissions, speed reducers, etc.

Tab. RUL-T.3.1 *Advantages and disadvantages, especially compared with sliding bearings [Pop, 2006]*

Advantages	Disadvantages
high efficiency ($\eta = 0.95 \dots 0.98$); much lower friction losses, low friction coefficients (0.0015...0.005, the lower values corresponding to ball bearings); small axial overall size (high load capacity per unit length); low costs (mass production in specialized companies, easy maintenance, low lubricant consumption); operation over a wide temperature range ($-273^{\circ}\text{C} \dots 600^{\circ}\text{C}$); high interchangeability and international standardization	large radial overall dimensions; reduced service life at high speeds; sensitive to high manufacturing accuracy and strict mounting conditions; unstable behavior under shocks and vibrations; improper operation in contaminated environments

RUL-T.4 CLASSIFICATION AND DESIGNATION OF ROLLING BEARINGS. TYPES OF BEARING ARRANGEMENTS

RUL-T.4.1 CLASSIFICATION OF ROLLING BEARINGS

Tab. RUL-T.4.1.1 Classification of rolling bearings [Jula, 1978]

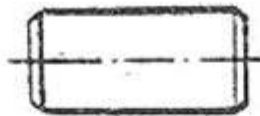
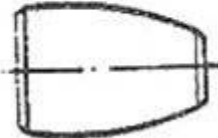
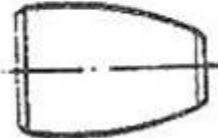
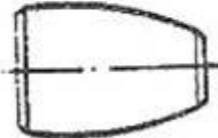
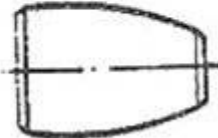
Criterion	Bearing type			
Shape of the rolling elements	Ball bearings			
	Roller bearings (cylindrical, needle, tapered, barrel)			
Number of rows of rolling elements	Single row			
	Double row			
	Multi-row			
Internal clearance (see fig. RUL-T.7.3)	Normal clearance			
	Reduced clearance			
	Increased clearance			
Manufacturing accuracy	Normal accuracy			
	High accuracy			
Direction of the load carried	Radial			
	Radial-axial (angular contact)			
	Axial-radial			
	Axial (thrust)			

Fig. RUL-T.4.1.1 Rolling elements of bearings: *a* – ball; *b* – cylindrical roller; *c* – needle; *d* – tapered roller; *e* – symmetrical barrel roller; *f* – asymmetrical barrel roller

Tab. RUL-T.4.1.2 Representative groups of rolling bearings

RADIAL bearings	
Deep groove ball bearings	

Main characteristics: carry **medium radial and small axial loads**; zero axial displacement in both directions; maximum misalignment 8 minutes

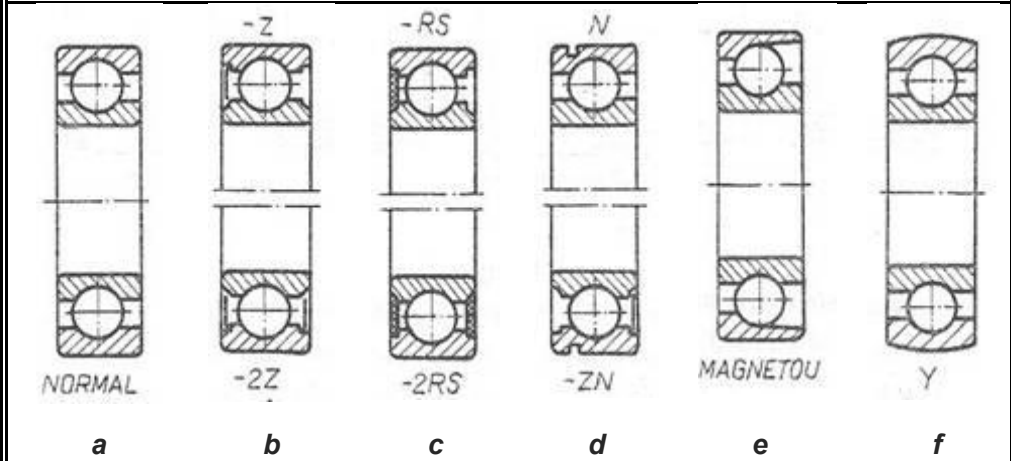


Fig. RUL-T.4.1.2 Single-row deep groove ball bearings: **a** – open; **b** – shielded on one side/both sides; **c** – sealed on one side/both sides; **d** – with snap ring groove/with a shield on one side and snap ring groove; **e** – magneto type (carry higher axial loads in one direction); **f** – Y type (allow free rotation of the outer ring in the housing and accommodate large misalignments)

Self-aligning radial bearings
Main characteristics: carry high radial and small-to-medium axial loads in both directions; zero axial displacement in both directions; increased misalignment, max. 3...5 degrees

Needle roller bearings
Main characteristics: carry high radial loads ; small misalignment; operation at low speeds; small radial overall size

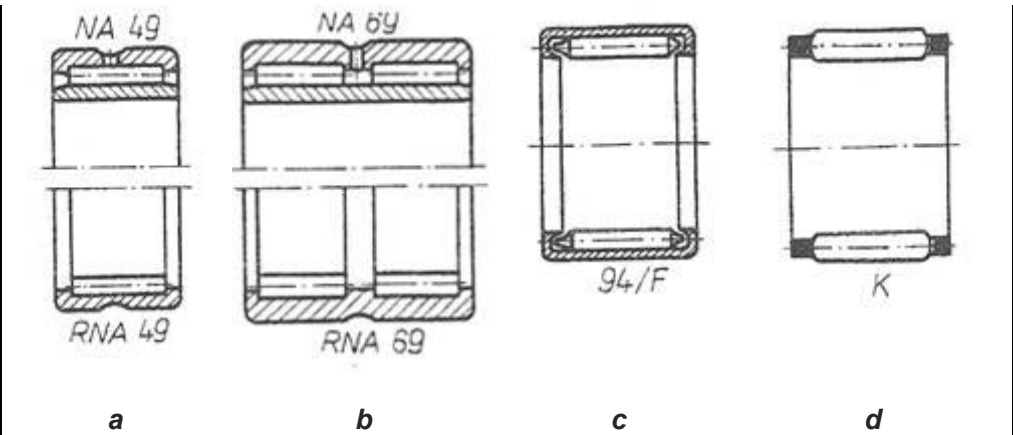
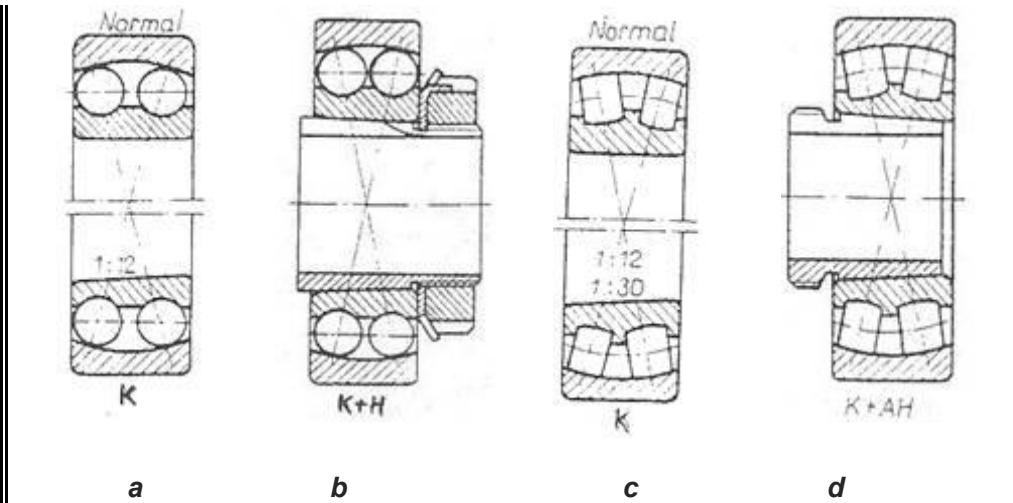


Fig. RUL-T.4.1.5 Needle roller bearings: **a** – single row (with two rings/without inner ring); **b** – double row (with two rings/without inner ring); **c** – drawn cup type; **d** – needle roller and cage assembly

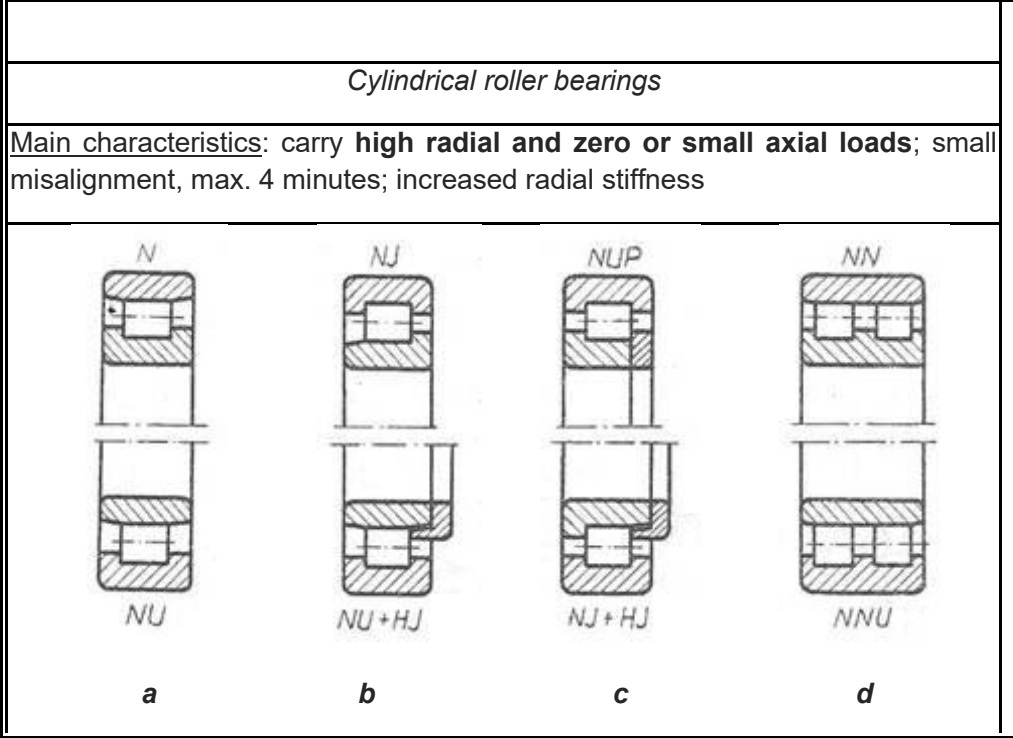
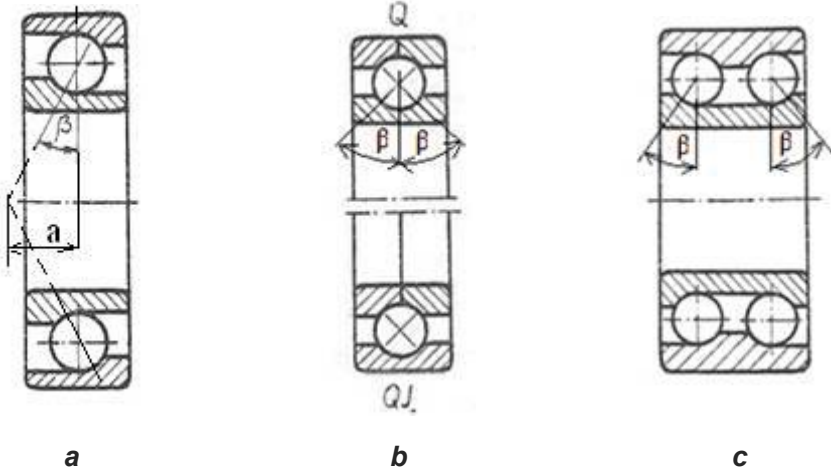
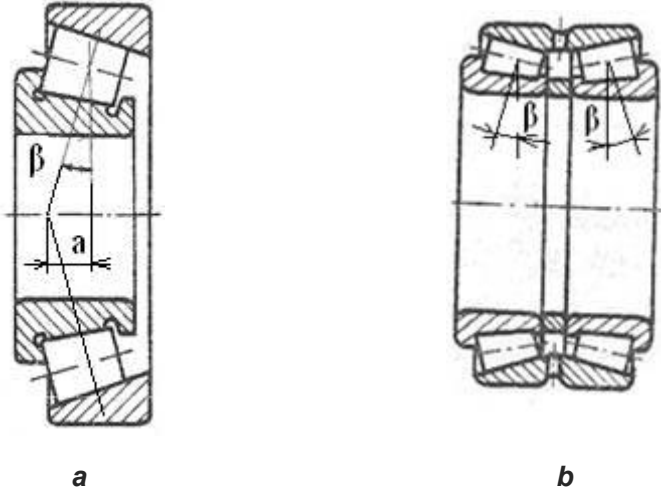


Fig. RUL-T.4.1.3 Cylindrical roller bearings: **a** – single row, with separable outer ring/separable inner ring (do not carry axial forces and allow axial displacements); **b** – single row, with one rib/flange on the inner ring (carry axial forces in one direction and prevent axial displacement in that direction); **c** – single row, with two ribs/flanges on the inner ring (carry axial forces in both directions and prevent axial displacements); **d** – double row, with separable inner/outer ring (do not carry axial forces and allow axial displacements)

RADIAL-AXIAL bearings

Angular contact ball bearings
Main characteristics: carry medium-to-high radial and medium axial loads
 a b c Fig. RUL-T.4.1.6 Angular contact ball bearings: a – single row (carry <u>axial forces in one direction only</u>; mounted in pairs, X, O or tandem arrangement); b – four-point contact (carry small radial and medium axial loads); c – double row

Tapered roller bearings
Main characteristics: carry high radial and high axial loads
 a b Fig. RUL-T.4.1.7 Tapered roller bearings: a – single row (carry <u>axial forces in one direction only</u>; mounted in pairs, X, O or tandem arrangement); b – double row

AXIAL-RADIAL bearings

AXIAL (thrust) bearings

Main characteristics: carry small radial and high axial loads

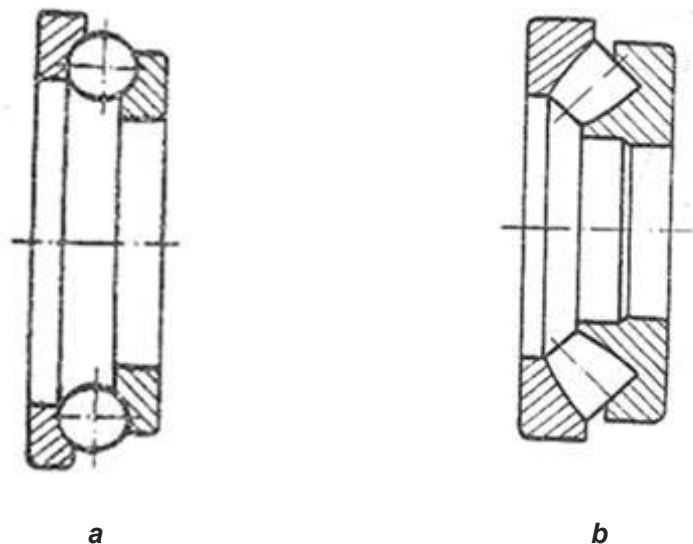


Fig. RUL-T.4.1.8 Axial-radial bearings: **a** – ball; **b** – asymmetrical barrel (spherical) roller

Main characteristics: carry high axial forces and do not carry radial forces

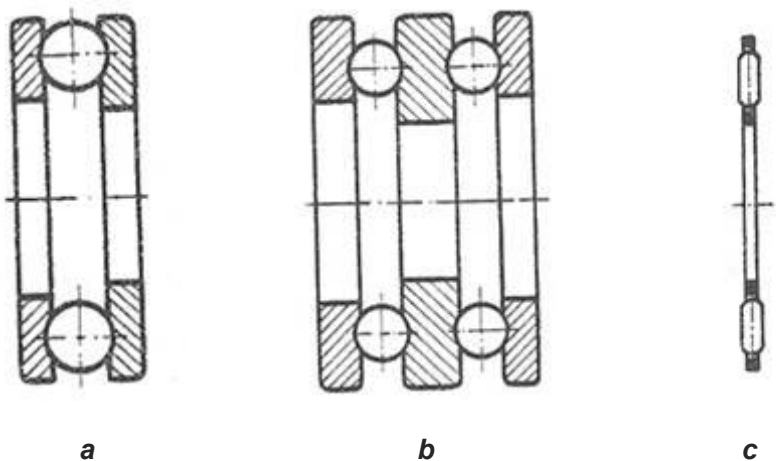


Fig. RUL-T.4.1.9 Thrust bearings: **a** – single-row ball (single direction, carry axial forces in one direction only); **b** – double-row ball (double direction, carry axial forces in both directions); **c** – needle roller and cage thrust assembly (for low speeds)

RUL-T.4.2 DESIGNATION OF ROLLING BEARINGS

Tab. RUL-T.4.2.1 Designation of rolling bearings

Generation of the dimension code (dimension series)				General structure of the designation
Codul lăţimii: 0 Codul diametrului: 303 02 00 13 12 10 23 22 20 33 32 30				Prefixes: alphabetic codes indicating information about materials (other than the usual ones). Suffixes: alphabetic codes indicating information about modifications compared with the standard design. Basic designation: type code (alphanumeric code of the bearing)

← Basic designation →					type: e.g. 6 – deep groove ball bearing, NU – cylindrical roller bearing with separable inner ring); <u>dimension series code</u> (one- or two-digit numerical code according to the adjacent figure); <u>bore code</u> (one digit from 1 to 9 – bearing with a bore of 1, 2, 3...9 mm; 01, 02, 03 – bearing with a bore of 12, 15 and 17 mm, respectively; 04, 05...99 – bearing with a bore diameter of 04×5, 05×5...99×5 mm)
Prefixes	Type code	Dimension series code	Bore code	Suffixes	
Note. Examples of basic designations: 6206 – single-row deep groove ball bearing (6), dimension series (2), bore diameter (30 mm); 32309 – single-row tapered roller bearing (3), dimension series (23), bore diameter (45 mm).					

RUL-T.4.3 TYPES OF BEARING ARRANGEMENTS

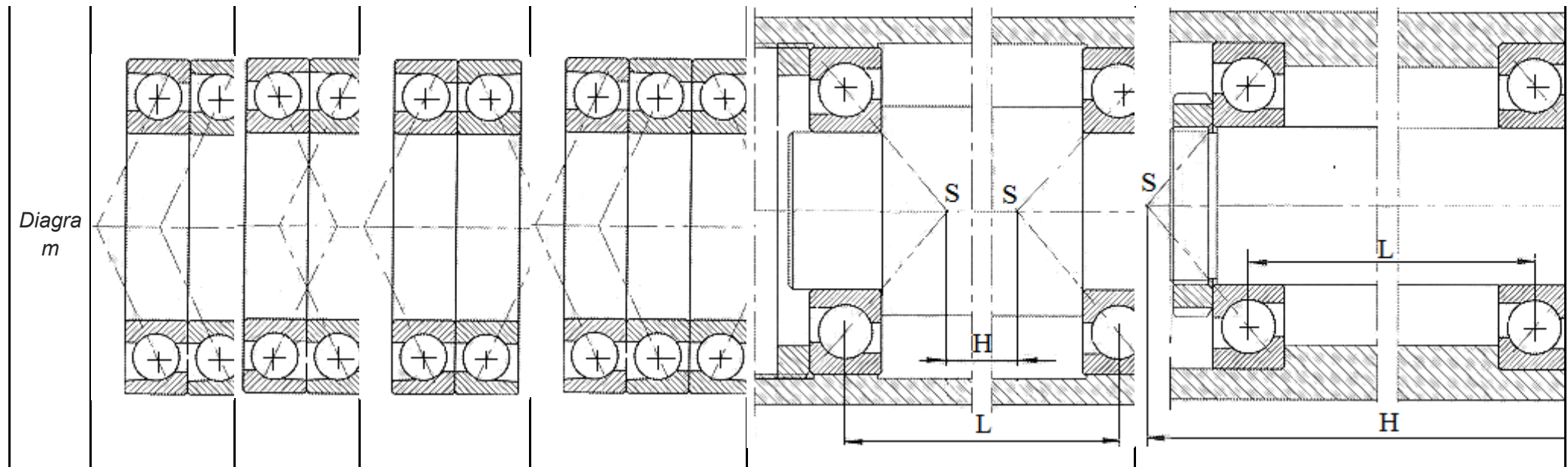
Rolling bearing arrangements of shaft supports have the following functions:

Main function: radial (centering) and axial location and support of the shaft, taking up the radial and axial forces.

Secondary functions: to allow free thermal expansion of the shaft as the operating temperature varies; to provide lubrication, sealing and, where necessary, cooling of the bearing.

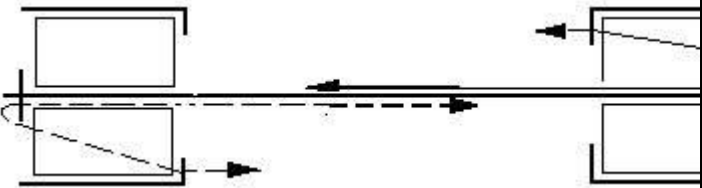
Tab. RUL-T.4.3.1 Possible combinations (arrangements) of rolling bearings in supports [xxxx, 2010]

Type	Within the same support				In two different supports	
	Tandem	X arrangement	O arrangement	Combined (e.g. tandem and X)	X arrangement	O arrangement



Tab. RUL-T.4.3.2 Types of rolling bearing arrangements

Type		Diagram	Main characteristics
With two supports	Axial location in both directions in a single locating bearing (the other bearing is axially free (non-locating), see fer. RUL-S.1)		Allows thermal expansion of the shaft in the non-locating bearing (the axial displacement can even take place inside the bearing of this support if it is separable). Recommended for long shafts and/or shafts operating under large temperature variations (significant thermal deformations). When single-row angular contact bearings are used, locating supports with X or O arrangements can be obtained.
	Axial location in the two supports, cross-located (each for one direction, see fer. RUL-S.1)		Thermal expansion of the shaft reduces/increases the bearing clearances as the operating temperature increases/decreases. Recommended for short shafts with insignificant thermal deformations. When single-row angular contact bearings are used, an X arrangement is obtained (the distance between the effective load centers is smaller than the distance between

				the bearings, $H < L$ (Tab. RUL-T.4.3.1); the arrangement increases the bending stiffness).
		direction outer-inner		Thermal expansion of the shaft, caused by the increase/decrease of the operating temperature, affects the bearing clearances depending on the distance between the outer cones: it reduces/increases them for the case in fig. RUL-T.4.3.2,a; it does not affect them for the case in fig. RUL-T.4.3.2,b; it increases/reduces them for the case in fig. RUL-T.4.3.2,c. Recommended for short and rigid shafts (insignificant bending deformations). When single-row angular contact bearings are used (fig. RUL-T.4.1.6,a; RUL-T.4.1.7,a), an O arrangement is obtained (the distance between the effective load centers is greater than the distance between the bearings, $H > L$ (Tab. RUL-T.4.3.1); the arrangement reduces the bending stiffness)

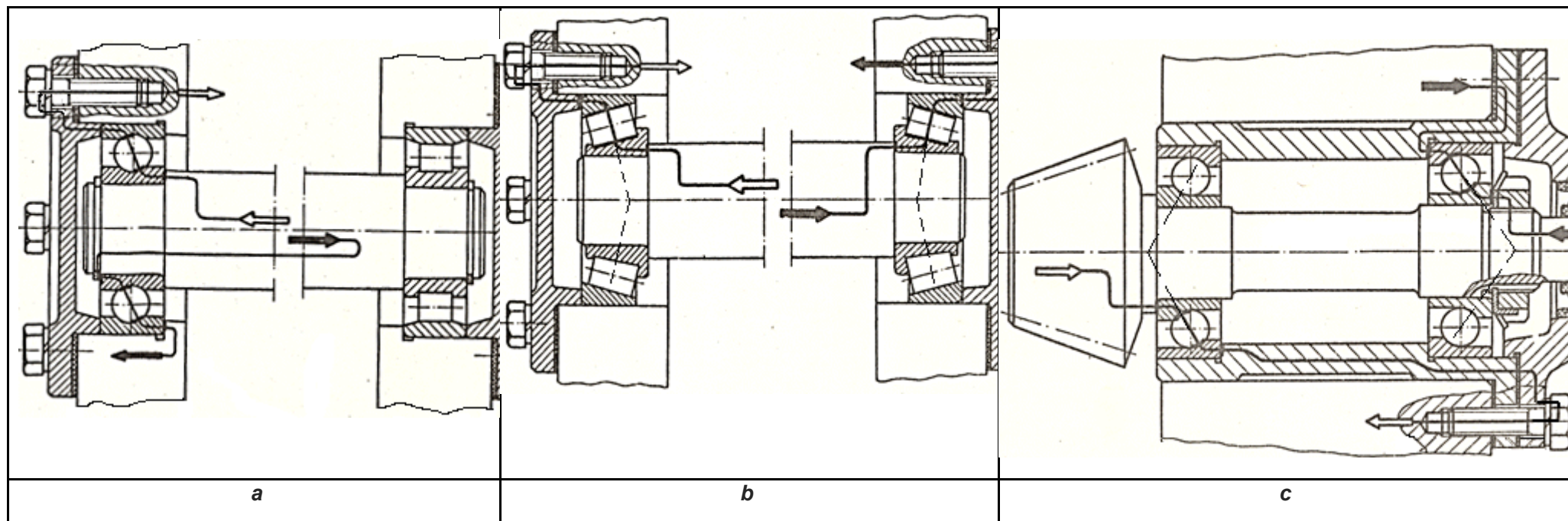


Fig. RUL-T.4.3.1: Design solutions of rolling bearing arrangements: *a* – locating bearing with a deep groove ball bearing and non-locating bearing with a cylindrical roller bearing; *b* – cross-located inner-outer supports with tapered roller bearings in X arrangement; *c* – cross-located outer-inner supports with angular contact ball bearings in O arrangement

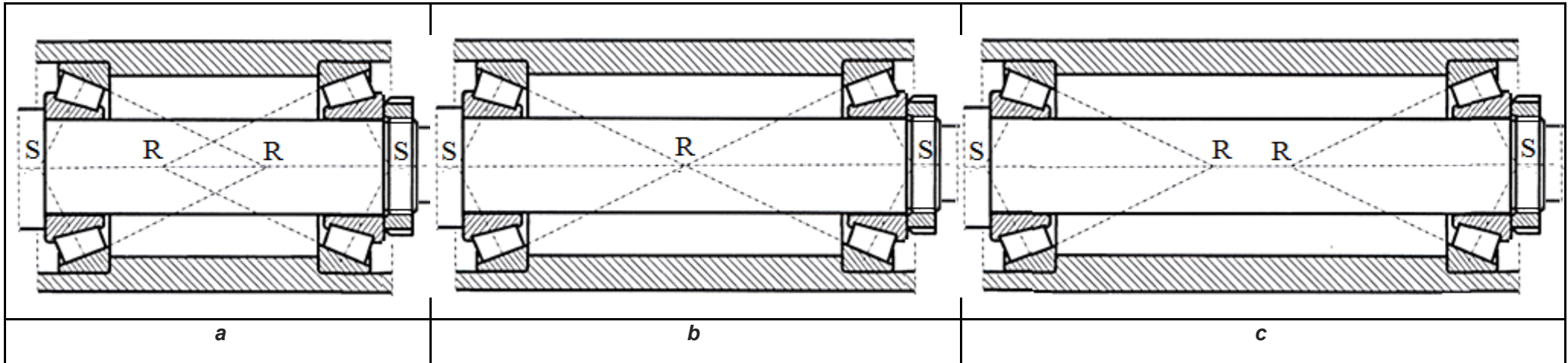


Fig. RUL-T.4.3.2: Variants of cross-located outer-inner arrangements with angular contact bearings in O arrangement: **a** – the apexes of the outer cones overlap; **b** – the apexes of the outer cones coincide; **c** – the apexes of the outer cones do not overlap (variants b and c are used mainly for bearings mounted in different supports)

RUL-T.5 MATERIALS AND TECHNOLOGIES

The main elements (rings and rolling elements) are made mainly of carbon steels (1% C) alloyed with chromium (1.3...1.65%), manganese and silicon, known as RUL1 or RUL2. After hardening to 62...65 HRC they achieve high toughness and increased resistance to contact stresses and wear. For bearings operating at high temperatures and in very humid environments, special high-alloy or corrosion-resistant steels are used, usually alloyed with chromium.

Bearing cages are made of anti-friction materials with high vibration damping capacity: mild carbon steels, for cages stamped from sheet metal; brass, bronze, textolite, plastics – for the solid (machined) cages of high-speed bearings.

Rolling bearings are produced in specialized companies based on the complex manufacturing technologies associated with the various bearing groups.

RUL-T.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. RUL-T.6.1 Forms and causes of improper behavior or removal from service [Danape, 2012]

Consequences	Forms	Occurrence	Causes	Manifestation	Avoidance
Improper behavior (with irregular motion, vibrations and noise)	Damage to the raceways and/or rolling elements by contact fatigue (pitting, spalling)	Mainly in rotating bearings with speeds $n > 10$ rpm, well lubricated and sealed	Fatigue of the surface layers of the raceways and/or rolling elements caused by time-varying contact stresses	Initial microcracks on the active surfaces, which grow as a result of the hydrostatic pressure of the oil generated during the contacts between the rolling elements and the raceways, followed by the detachment of small particles of material, resulting in small pits (pitting, fig. RUL-T.6.1); the growth of the pits over time leads to increased clearance and improper operation with vibrations and noise	Limiting the operating time by <u>life calculation</u> based on the <u>dynamic load rating</u>
	Local plastic deformation of the raceways	Heavily loaded bearings at very low speeds ($n < 10$ rpm), stationary or with slow oscillating motions, lubricated with unsuitable lubricants	Exceeding the yield strength of the material	Permanent indentations on the raceways; irregular motion	Limiting the loads by <u>static calculation</u> based on the <u>static load rating</u>
	Abrasive wear of the raceways and/or rolling elements	Bearings operating in environments with dust and abrasive impurities, with inadequate sealing	Exceeding the wear limit of the material	Scratches on the active surfaces; increased bearing clearance and, therefore, improper operation with vibrations and noise	Increasing the effectiveness of the sealing and lubrication systems
Removal from service	Seizure	Heavily loaded bearings operating at high temperatures and/or with insufficient lubrication	Local decrease of the contact strength of the material	Local micro-welds between the raceways and the rolling elements and/or between the cage and the rolling elements; loss of running accuracy, high noise, increase of the rotational resistance up to locking	Increasing the effectiveness of the lubrication system

	Cage failure	Bearings operating at high speeds under combined loads and/or improperly mounted	Increased centrifugal forces, self-excited vibrations	Wear of the cage in the contact areas with the rolling elements; reduction of the cross-sectional areas, cracking and breaking of the cage	Proper mounting and operation
 The figure consists of three black and white photographs illustrating damage by pitting in rolling bearings. The top left photograph shows a cross-section of a bearing ring with a rough, pitted surface. The top right photograph shows a bearing cage with pitting on its contact surfaces. The bottom photograph shows several individual rolling elements with visible pitting damage.					
Fig. RUL-T.6.1 Damage by pitting (sources: Collins, J.A., <i>Mechanical Design of Machine Elements and Machines</i>; Cornet, 1998)					

RUL-T.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

Main functional parameters:

The life of a rolling bearing is the number of revolutions performed by the rotating ring until the first signs of material fatigue appear (at constant speed it can be expressed in hours and is also called the operating life).

The reliability (life) of a batch of rolling bearings, L , is the number of revolutions reached or exceeded by a percentage of the bearings of a test batch without signs of material fatigue (pitting); by convention, the basic rating life L_b is defined as one million revolutions, reached by 90% of the bearings of a batch without pitting damage (fig. RUL-T.7.1).

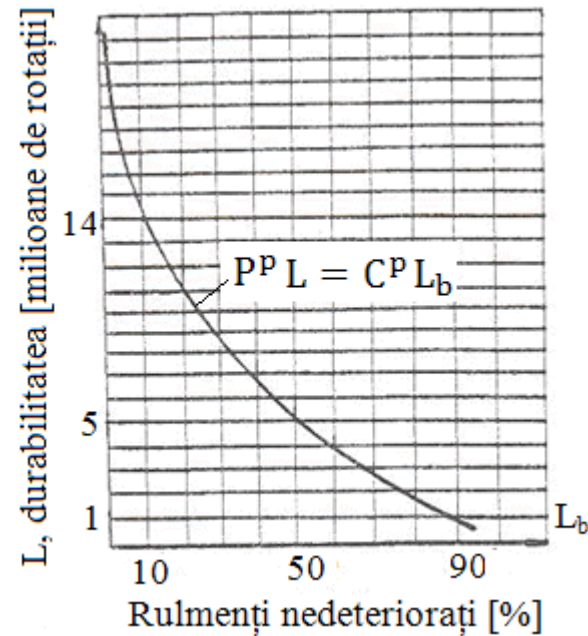


Fig. RUL-T.7.1 Life (fatigue) curve

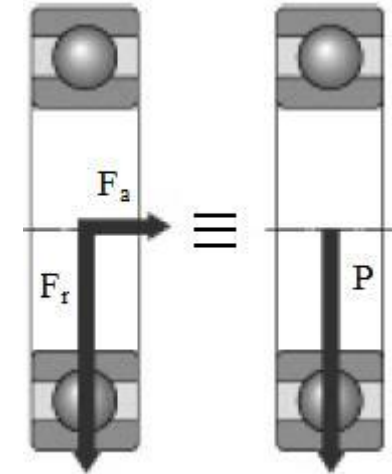


Fig. RUL-T.7.2 Load equivalence

The basic dynamic load rating, **C**, is the radial load for radial and radial-axial bearings, or the axial load for axial and axial-radial bearings, of constant magnitude and direction, which, acting on a batch of apparently identical bearings, provides the basic rating life, **L_b** (one million revolutions).

The equivalent dynamic load, **P**, is the purely radial load for radial or radial-axial bearings (or purely axial, for axial or axial-radial bearings), of constant magnitude and direction, under which a bearing reaches the same life as under the actual loading conditions with combined radial and axial forces (fig. RUL-T.7.2).

The basic static load rating, **C₀**, is the purely radial load for radial or radial-axial bearings (or purely axial, for axial and axial-radial bearings) that produces in the contact area a plastic (permanent) deformation with a depth of 0.0001 of the rolling element diameter.

The equivalent static load, **P₀**, is the radial load for radial and radial-axial bearings (or the axial load, for axial bearings) that produces the same plastic (permanent) deformation as the loading with combined (radial and axial) loads.

Other functional parameters:

The internal clearance (fig. RUL-T.7.3) is the radial displacement (radial clearance), axial displacement (axial clearance) or angular displacement (tilting) possible without deformation of the bearing parts; with respect to radial clearance, bearings are manufactured in the classes: C1 and C2 (small clearances); C0 (or CN, normal clearances); C3, C4, C5 (large clearances).

The limiting speed is the speed a bearing can reach under design and operating conditions rigorously defined in the manufacturers' catalogues.

The manufacturing accuracy is expressed by tolerance classes: P0 – normal tolerance class; P6, P5, P4 and P2 – higher tolerance classes for bearings with very precise applications and for use at high speeds.

The frictional moment is a functional parameter quantifying the energy required for rotation under load and at start-up (starting moment). Manufacturers give the relations and parameters for calculating these moments.

The operating temperature is specified by the manufacturer depending on the bearing type and the lubricant.

The stiffness is a global parameter that is rarely given in the catalogues of common bearings.

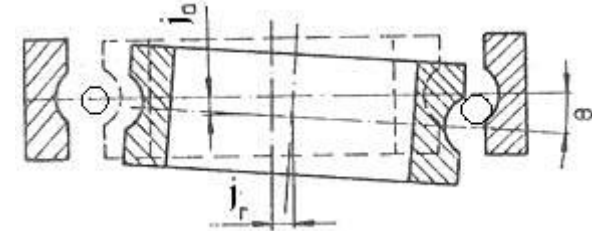


Fig. RUL-T.7.3 Bearing internal clearances

Main constructive parameters (fig. RUL-T.7.4): d – bore diameter; D – outside diameter; B , T – width; α – contact angle ($\alpha = 0^\circ$ for radial bearings; $0^\circ < \alpha < 45^\circ$ for radial-axial bearings; $45^\circ < \alpha < 90^\circ$ for axial-radial bearings; $\alpha = 90^\circ$ for axial bearings); a – position of the pressure cone apex; C – ring width.

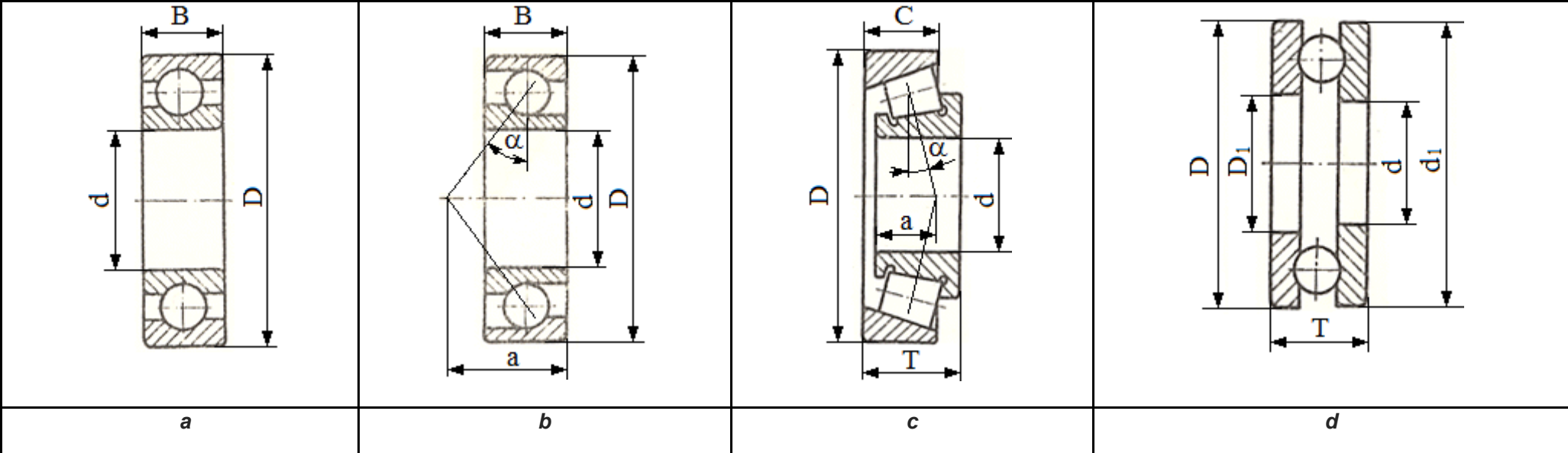


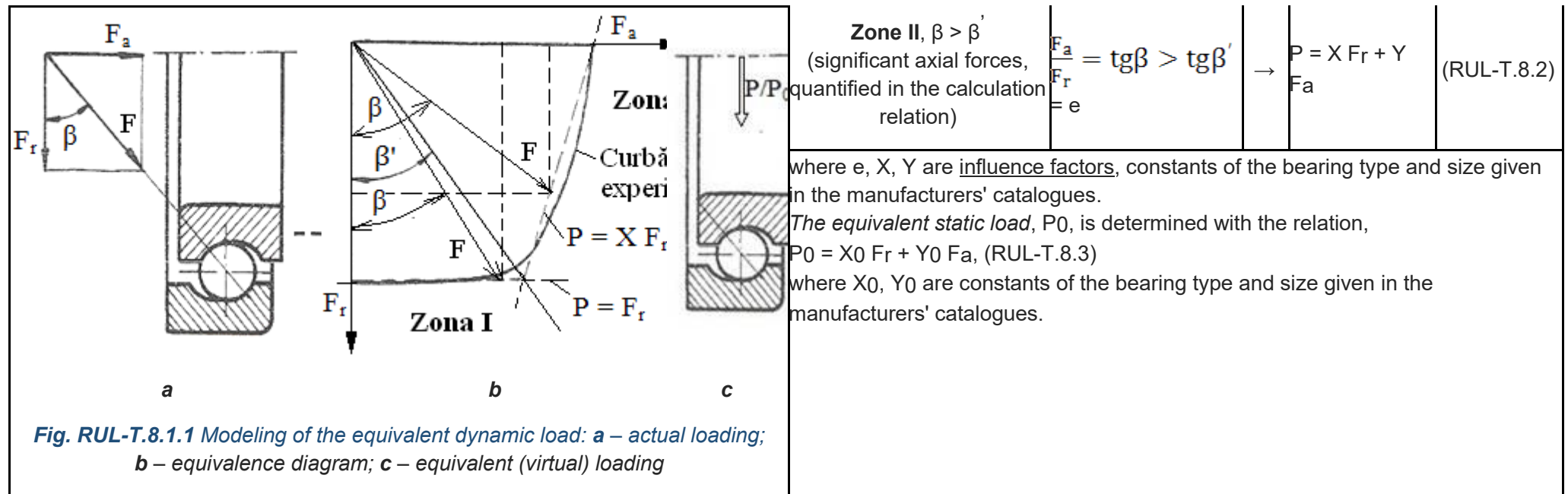
Fig. RUL-T.7.4 Main constructive parameters (examples): a – deep groove ball bearing; b – angular contact ball bearing; c – tapered roller bearing; d – thrust ball bearing

RUL-T.8 CALCULATION MODELS

RUL-T.8.1 CALCULATION MODELS OF BEARING LOADS

RUL-T.8.1.1 Calculation models of the equivalent dynamic/static loads of bearings with constant speeds and external loads

The equivalent dynamic load P, obtained by approximating the experimental curve in fig. RUL-T.8.1.1,b with two straight lines (in zones I and II), is determined with the relations:			
Zone I, $\beta \leq \beta'$ (small, negligible axial forces)	$\frac{F_a}{F_r} = \text{tg}\beta \leq \text{tg}\beta'$ $= e$	→	$P = F_r$ (RUL-T.8.1)



RUL-T.8.1.2 Calculation models of the loads of angular contact bearings in X or O arrangement

For angular contact bearings (ball or roller), in which the force is transmitted obliquely at the contact angle β (fig. RUL-T.8.1.1,a), the radial force generates an internal (induced) axial force, $F'_a = 0,5 F_r / Y$, which is evaluated for each support (A and B) and affects the axial loading of the supports. Fig. RUL-T.8.1.2.1,a,b and tab. RUL-T.8.1.2.1 present the diagrams and, respectively, the relations of the loading forces of shafts and angular contact bearings in X and O arrangements.

Note. If only the internal (induced) force acts in the angular contact bearing, since the ratio F'_a / F_r is always smaller than the factor e , the equivalent dynamic load is $P = F_r$.

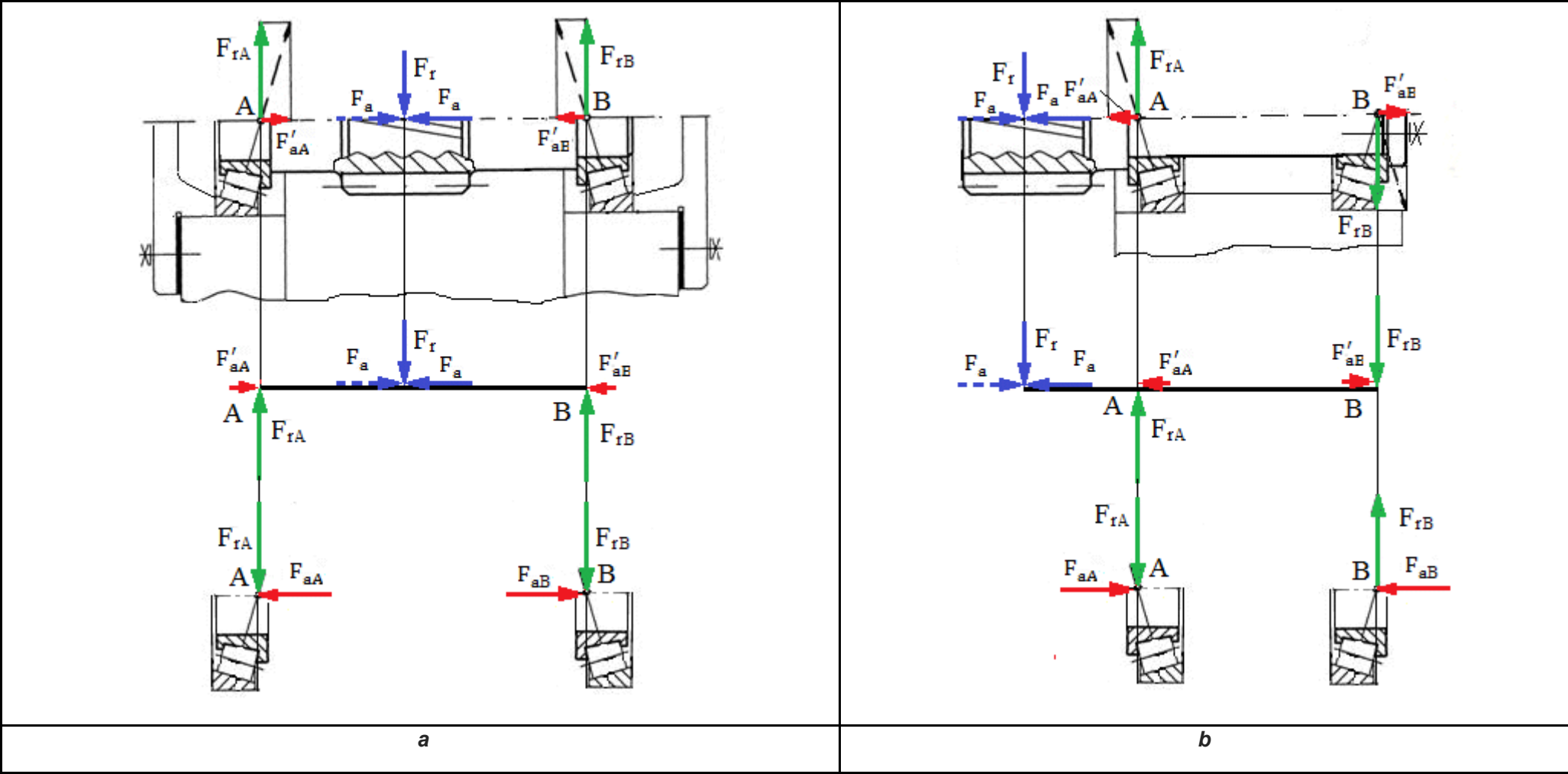


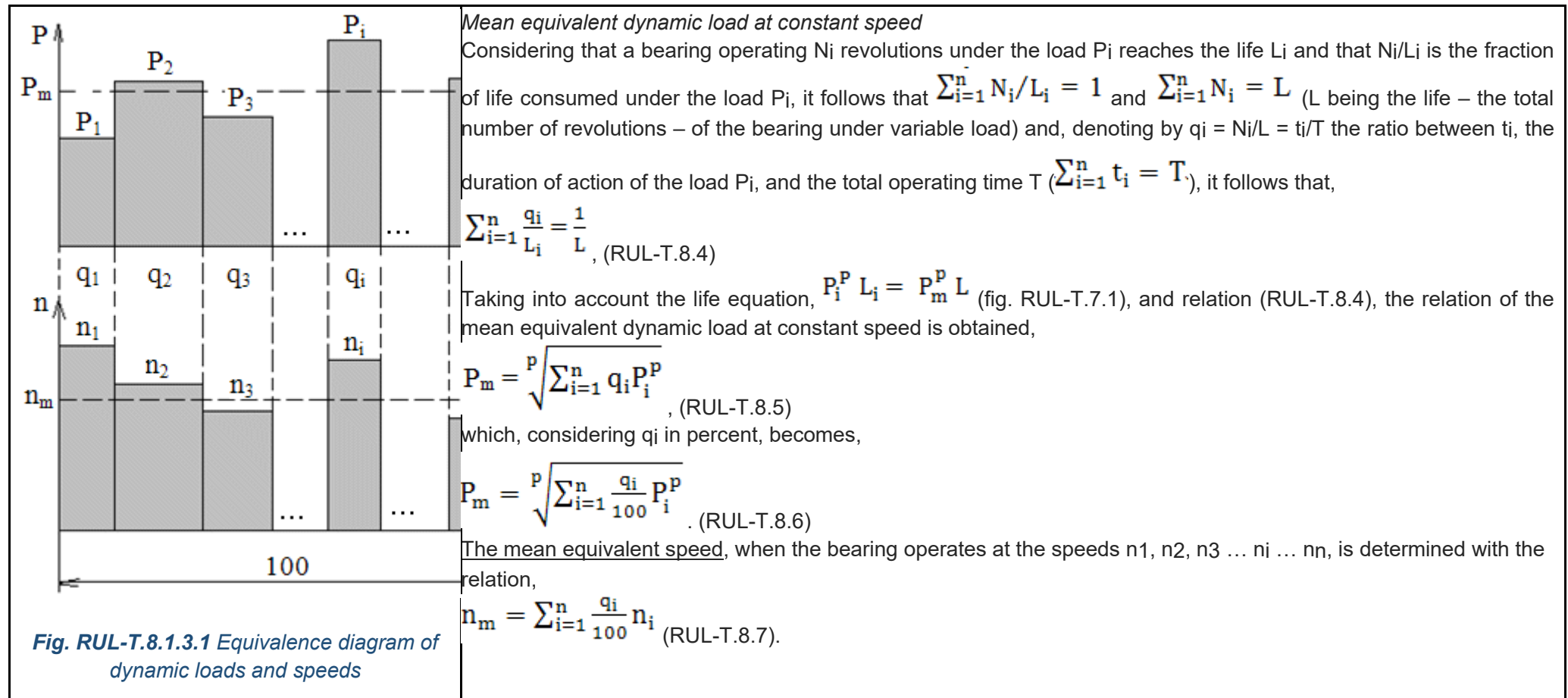
Fig. RUL-T.8.1.2.1 Calculation diagrams of the axial forces in paired angular contact bearings: **a** – X arrangement; **b** – O arrangement

Tab. RUL-T.8.1.2.1 Calculation relations of the loading forces of angular contact bearings

Direction of the axial force F_a	X arrangement (fig. RUL-T.8.1.2.1,a)			O arrangement (fig. RUL-T.8.1.2.1,b)		
	Shaft loading condition	Axial force loading the bearing		Shaft loading condition	Axial force loading the bearing	
		A	B		A	B

B → A	$F_a + F'_{aB} - F'_{aA} > 0$	$F_{aA} = F_a + F'_{aB}$	$F_{aB} = F'_{aB}$	$F_a + F'_{aA} - F'_{aB} > 0$	$F_{aA} = F'_{aA}$	$F_{aB} = F_a + F'_{aA}$
	$F_a + F'_{aB} - F'_{aA} < 0$	$F_{aA} = F'_{aA}$	$F_{aB} = F'_{aA} - F_a$	$F_a + F'_{aA} - F'_{aB} < 0$	$F_{aA} = F'_{aB} - F_a$	$F_{aB} = F'_{aB}$
A → B	$F_a + F'_{aA} - F'_{aB} > 0$	$F_{aA} = F'_{aA}$	$F_{aB} = F_a + F'_{aA}$	$F_a + F'_{aB} - F'_{aA} > 0$	$F_{aA} = F_a + F'_{aB}$	$F_{aB} = F'_{aB}$
	$F_a + F'_{aA} - F'_{aB} < 0$	$F_{aA} = F'_{aB} - F_a$	$F_{aB} = F'_{aB}$	$F_a + F'_{aB} - F'_{aA} < 0$	$F_{aA} = F'_{aA}$	$F_{aB} = F'_{aA} - F_a$

RUL-T.8.1.3 Calculation models of the equivalent dynamic loads of bearings with variable (stepped) speeds and external loads



	The mean equivalent dynamic load, when the bearing is loaded with the dynamic loads P1, P2, P3 ... Pi ... Pn at the speeds n1, n2, n3 ... nj ... nn, respectively, is determined with the relation, $P = \sqrt[p]{\sum_{i=1}^n \frac{q_i}{100} \frac{n_i}{n_m} P_i^p} \quad . \text{ (RUL-T.8.8)}$
--	---

RUL-T.8.2 LIFE AND STATIC CALCULATION MODELS OF ROLLING BEARINGS

Life calculation	
The basic life equation of a rolling bearing, associated with the experimental curve in fig. RUL-T.7.1 (see subchapter RUL-T.7), is $P^p L = C^p L_b, \text{ (RUL-T.8.7)}$ where p is the exponent of the fatigue curve (p = 3 for ball bearings; p = 10/3 for roller bearings), L – the life in millions of revolutions, P – the equivalent dynamic load (see subchapter RUL-T.7), C – the dynamic load rating and Lb = 1 million revolutions – the basic rating life	Sizing calculation, $C_{nec} = P \sqrt[p]{L} \quad . \text{ (RUL-T.8.8)}$ Verification calculation, $L_{ef} = \left(\frac{C}{P}\right)^p \geq L_{nec} \quad \text{ (RUL-T.8.9)}$ Calculation of the allowable load, $P_{cap} = \frac{C}{\sqrt[p]{L}} \quad \text{ (RUL-T.8.10)}$
Static calculation	
Usually, a <i>verification calculation</i> is performed with the relation C0 > P0 (RUL-T.8.11)	

RUL-T.9 MOUNTING AND OPERATION [JULA, 1978; XXXX, 2010]

Lubrication of rolling bearing supports	
<u>Purpose</u>: reduction of friction; corrosion protection; heat removal; noise reduction. <u>Lubricants used</u>: mineral oils or greases (fer. RUL-P.5). <u>Oil lubrication systems</u> (fer. RUL-S.2): own oil bath, oil circulation, oil injection, splash lubrication, drip lubrication, oil mist; grease lubrication.	
Fits, manufacturing tolerances and surface roughness of the shaft journal and of the housing bore	
<u>Purpose</u>: secure fixing of the rings, easy mounting and dismounting.	

Types of fits: shaft – inner ring: hole-basis system; outer ring – housing: shaft-basis system; the tolerance zones are determined mainly by the loads and by the equipment in which the support is integrated (fer. [RUL-P.3](#)).

Form and position deviations are specified for the shaft and the housing bore (fer. [RUL-P.4](#)).

Surface roughness is specified for the shaft and the housing bore (fer. [RUL-P.4](#)).

Axial fixing, mounting and dismounting of rolling bearings

The axial fixing of the inner ring on the shaft and of the outer ring in the housing aims to transmit the axial forces and/or prevent axial displacement (fer. [RUL-S.3](#)).

Mounting is done by pressing at room temperature with suitable tools and devices, or by hot mounting for bearings with $d > 50$ mm (fer. [RUL-S.4](#)); mounting sequence: first the rings with the tighter fit, then the one with the looser fit or with clearance.

Dismounting is done with suitable tools and devices in the reverse order of mounting (first the rings with the looser fit, then those with the tighter fit) (fer. [RUL-S.4](#)).

Clearance adjustment and preloading of rolling bearings

Purpose: ensuring an optimal clearance in the mounted state, for proper operation (low friction, no vibrations, increased guiding accuracy).

Clearance adjustment schemes (fer. [RUL-S.1](#)): by displacing the inner ring; by displacing the outer ring.

Sealing of rolling bearing supports

Purpose: protecting the bearing against the ingress of external impurities (dust, metal particles, water, etc.) and, on the other hand, preventing the lubricant from leaking out of the bearing housing.

Sealing systems (fer. [RUL-S.5](#)): bearing integral seals, non-contact or contact; seals at the support level, contact or non-contact.

Fer. [RUL-S.0](#) presents design solutions of rolling bearing arrangements specific to motor vehicle transmissions.

LGA. SLIDING BEARINGS

LGA-T. CONSTRUCTION AND MODELING OF SLIDING BEARINGS

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LGA-T.1 DEFINITION

Sliding bearings are subassemblies that materialize the kinematic pairs which ensure the support and/or guidance of the moving elements of machines (shafts, axles) by means of contact surfaces between which sliding friction occurs.

LGA-T.2 CONSTRUCTIVE STRUCTURE

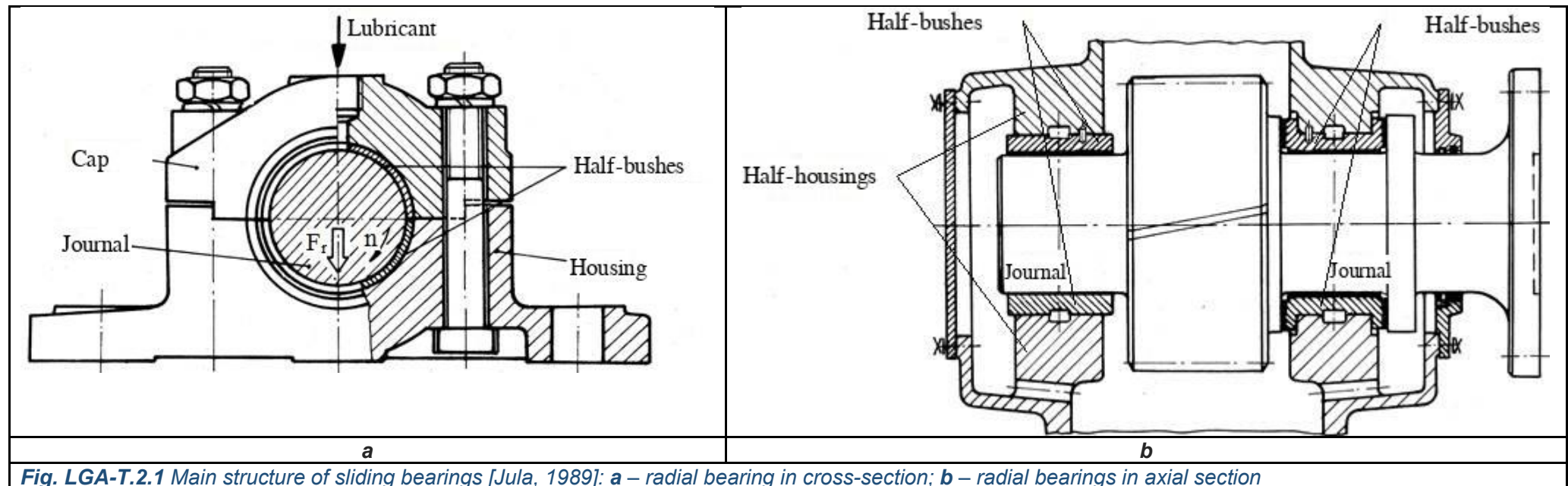


Fig. LGA-T.2.1 Main structure of sliding bearings [Jula, 1989]: **a** – radial bearing in cross-section; **b** – radial bearings in axial section

LGA-T.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Areas of use

Compared to rolling bearings, sliding bearings are used less frequently, in specific fields and conditions:

- high specific loads (e.g. over 2 MN) with significant shocks and vibrations;
- very high speeds (e.g. $n > 10000$ rpm), at which the durability of rolling bearings is reduced;

- very precise guidance is required;
- very large radial overall dimensions are required, for which rolling bearings are not manufactured in series;
- the cost of a sliding bearing is lower than that of a rolling bearing;
- in humid conditions and aggressive environments, in which rolling bearings cannot be used;
- in slow-running (e.g. $n < 10$ rpm) and lightly loaded mechanisms.

Tab. LGA-T.3.1 Advantages and disadvantages of sliding bearings [Rădulescu, 1985]

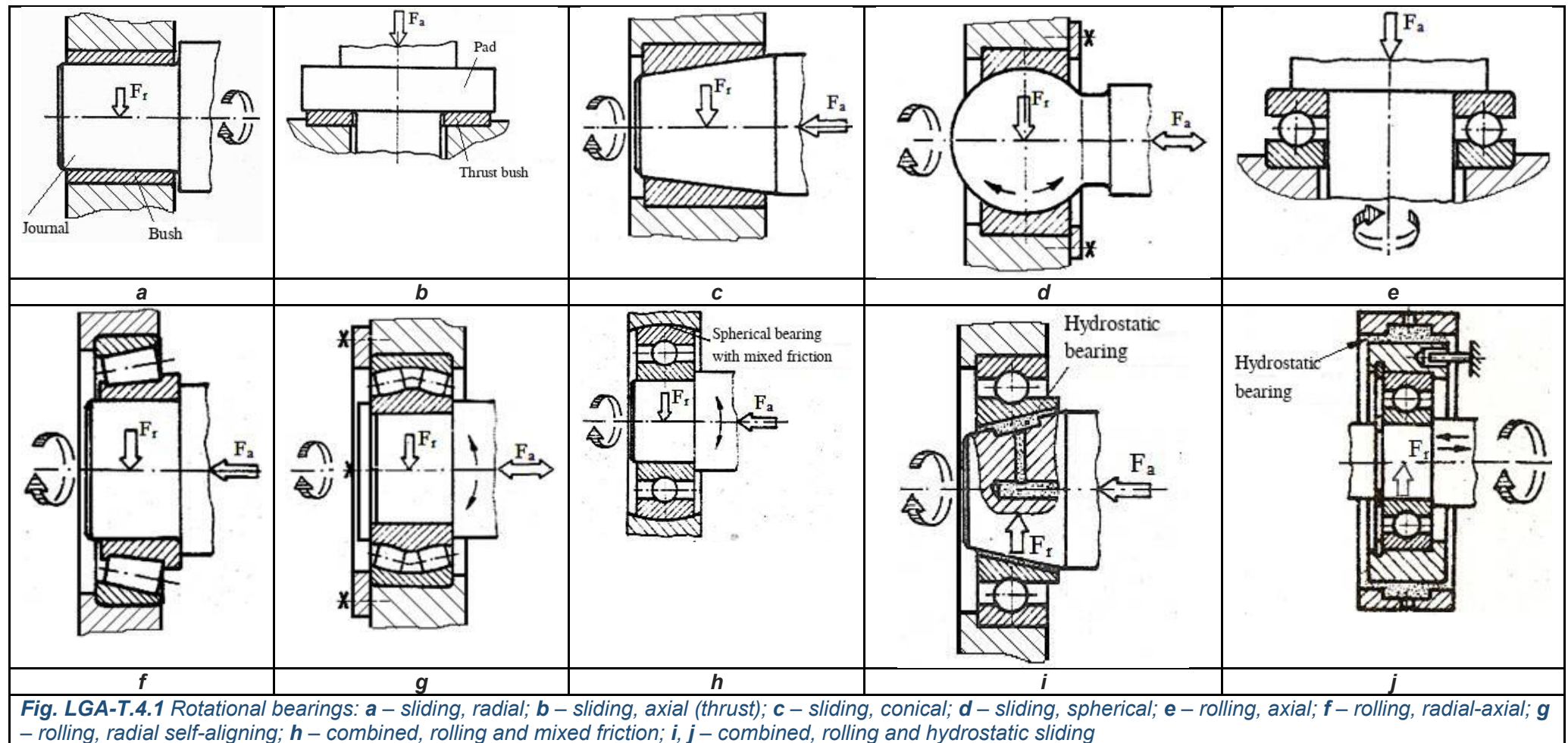
Advantages*	Disadvantages*
- operation at very low relative speeds, at which the use of rolling bearings can lead to permanent local deformations of the raceways; - high wear resistance; - they can operate at high and very high speeds, at which rolling bearings would have very low durability; - operation with low vibration and noise; - small radial overall dimensions; they can be made in very small sizes; - they allow kinematic pairs whose elements cannot be mounted axially (crankshafts)	- higher friction losses, especially at start-up - larger overall size in the axial direction - sometimes insufficient centering of the elements forming the kinematic pair - relatively imprecise guidance due to the relatively large clearance between journal and bearing bush - lower degree of standardization - high lubricant consumption - relatively complicated constructions - wear of the contact surfaces of the kinematic pair can lead to the replacement or repair of complex parts (shafts)
* Compared to rolling bearings	

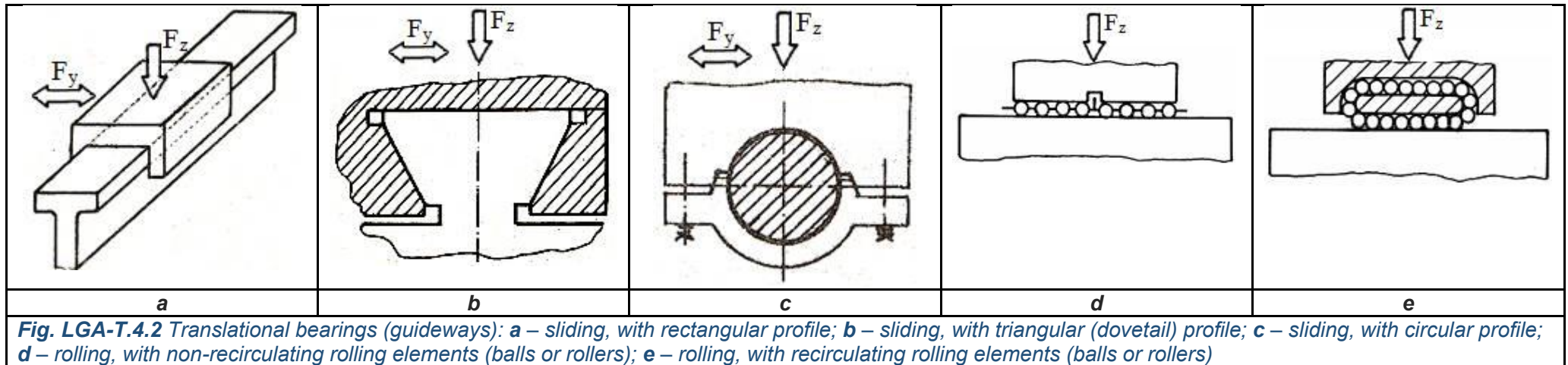
LGA-T.4 CLASSIFICATION

Tab. LGA-T.4.1 Classification of bearings

Criterion	Type	Scheme
Type of relative motion in the pair	Rotational	Fig. LGA-T.4.1
	Translational (guideway)	Fig. LGA-T.4.2
Type of relative motion	With sliding	Fig. LGA-T.4.1,a...d; Fig. LGA-T.4.2,a,b,c
	With rolling	Fig. LGA-T.4.1,f,g,h; Fig. LGA-T.4.2,d,e
	Hybrid (with sliding and rolling)	Fig. LGA-T.4.1,i,j
Type of lubricant	Lubricated with oil or grease	Fig. LGA-T.4.1; Fig. LGA-T.4.2
	Gas lubricated	
Friction regime	With boundary or mixed friction	Fig. LGA-T.4.1,a...h; Fig. LGA-T.4.2,a,b,c
	With fluid friction	Fig. LGA-T.4.1; Fig. LGA-T.4.2
Way of achieving fluid friction	Hydrodynamic (HD) or gas-dynamic (GD)	Fig. LGA-T.4.1,a
	Hydrostatic (HS) or gas-static (GS)	Fig. LGA-T.4.1,i,j
	Elastohydrodynamic (EHD)	Fig. LGA-T.4.1,f...i; Fig. LGA-T.4.2,d,e

	Hybrid	<i>Fig. LGA-T.4.1,i,j</i>
Shape of the friction surface	Cylindrical	<i>Fig. LGA-T.4.1,a</i>
	Conical	<i>Fig. LGA-T.4.1,c</i>
	Spherical	<i>Fig. LGA-T.4.1,d</i>
	Plane	<i>Fig. LGA-T.4.2,a,b,d,e</i>
Direction of the force	Radial	<i>Fig. LGA-T.4.1,a,c...j</i>
	Axial (thrust)	<i>Fig. LGA-T.4.1,b,e</i>
	Radial-axial	<i>Fig. LGA-T.4.1,c,d,f,i</i>
	Axial-radial	
Position on the shaft or axle	End (journal) bearings	<i>Fig. LGA-T.4.1,a,c,e</i>
	Intermediate	<i>Fig. LGA-T.4.1</i>
Type of support	Rigid	<i>Fig. LGA-T.4.1</i>
	Elastic	
	Self-aligning (oscillating)	<i>Fig. LGA-T.4.1,h</i>





LGA-T.5 MATERIALS AND TECHNOLOGIES

Antifriction materials for bearing bushes

Characteristics: high wear resistance; low coefficient of friction, to reduce friction losses; capacity to accept local plastic deformations; high contact fatigue strength; scuffing (seizure) resistance; high thermal conductivity; low coefficient of thermal expansion; adherence of the lubricant; corrosion resistance.

Ferrous metallic materials for bearing bushes: cast irons – grey Fc, with nodular graphite Fgn, malleable Fma or manganese Fmn; they are obtained by casting; they are used for solid bushes with abundant lubrication; they are not suitable for high speeds and behave poorly under shock loads.

Non-ferrous metallic materials for bearing bushes: bronzes – CuSn, CuPbSn, CuPb; Cu alloys with Sn and Pb are used for multilayer bushes obtained by casting, and Al alloys – AlSn, AlPb, AlCuPb – for multilayer bushes obtained by cladding; they are used in the construction of solid bushes for high loads and speeds.

Non-metallic materials for bearing bushes: *textolite* – as solid bushes it is corrosion resistant (water lubrication) and damps shocks; *wood* – oak – pressed and impregnated; *plastics* – bakelites (thermosetting), for solid bushes, and Teflon or polyamides – with self-lubricating properties; *rubber* – vulcanized multilayer bush (water lubrication); *graphite*; *ceramics* – solid bushes made of fine ceramics; *precious stones* – in precision mechanics;

Sintered materials can be: *soft*, based on FeC, FeCu, CuPb, AlCuPb, used for solid, porous and self-lubricating bushes made as thin multilayers; *hard*, based on metal carbides, used for gas bearings.

Composite materials: metal backing of steel strip, a sintered layer (bronze) and a solid lubricant.

Antifriction materials for journals (pads)

In most cases, the *journals* (pads) are made of the same material as the axle or shaft they belong to, mainly case-hardening or quenched and tempered carbon steels. To the requirements imposed on shaft and axle materials are added: high wear resistance; appropriate surface finishing.

It is recommended that the hardness of the journal be 3...5 times greater than that of the bush; this ratio ensures proper operation even when lubrication is deficient. For bush materials with hardness < 300 HB, hardening of the journal is no longer necessary. The more important unhardened journals are vibro-burnished, while journals hardened by quenching or by case hardening and quenching must be ground.

LGA-T.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

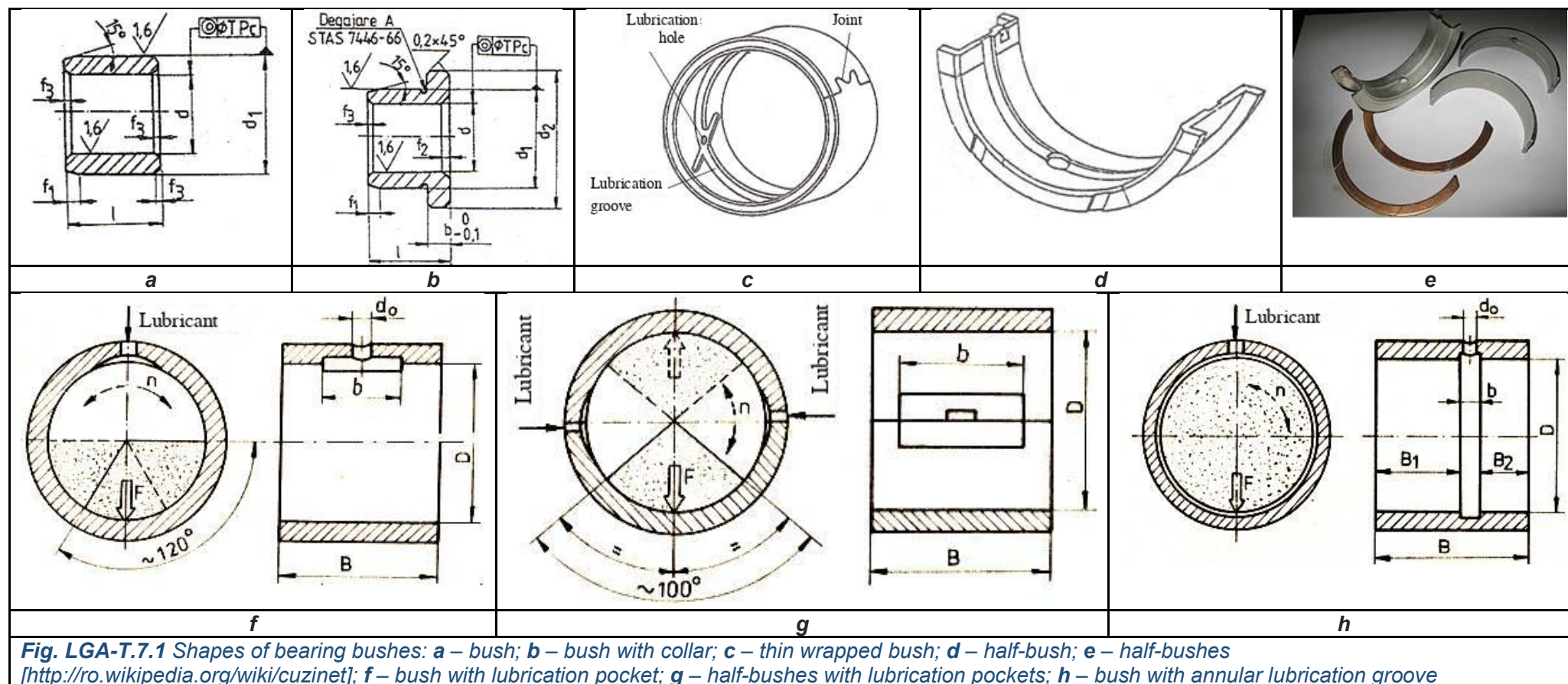
Tab. LGA-T.6.1 *Forms and causes of improper behavior or removal from service*

<i>Forms</i>	<i>Consequences</i>	<i>Occurrence</i>	<i>Causes</i>	<i>Manifestation</i>	<i>Avoidance</i>
Damage of the active surfaces	Improper behavior	Mainly in bearings with semi-dry lubrication	Wear	Non-uniform motion with vibration and noise	Limiting wear
Scuffing (seizure)	Removal from service	Mainly in bearings with improper lubrication at high loads and speeds	Catastrophic wear	Interruption of the relative motion (locking)	Limiting the temperatures and the maximum static or fatigue stresses to permissible values; Ensuring proper lubrication

LGA-T.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

Tab. LGA-T.7.1 *Shapes and functional parameters of bearing bushes*

<i>Shape</i>	<i>Type</i>	<i>Parameters</i>	<i>Description, uses</i>
<i>Bush</i>	Solid, thick-walled		Used in the construction of non-dismountable radial bearings, usually at the shaft end;
	Solid, thick-walled, with collar	d, l, d ₁ , b	Simple manufacturing technologies – machining of a cast or rolled blank; low deformability
	<i>Wrapped, thin-walled</i>		
<i>Half-bush (half shell)</i>			



LGA-T.8 CALCULATION MODELS

LGA-T.8.1 CALCULATION MODELS (SIMPLIFIED) OF BEARINGS WITH DRY, BOUNDARY OR MIXED FRICTION

LGA-T.8.1.1 Simplified calculation model of radial bearings

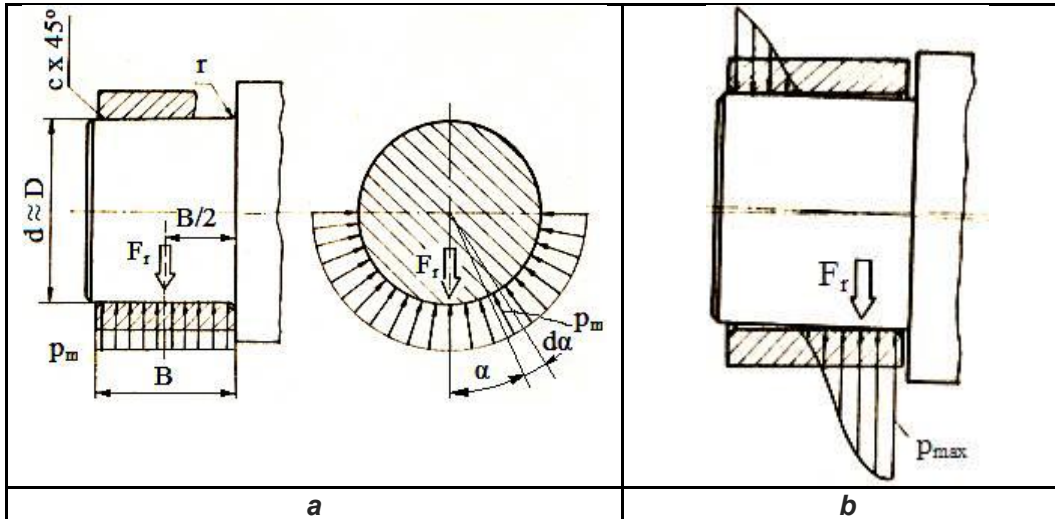


Fig. LGA-T.8.1.1.1 Simplified calculation model of the radial bearing: **a** – calculation scheme; **b** – axial section with coaxiality deviations

Assumptions:

- the journal is considered a straight beam fixed in the adjacent section of the shaft;
- the contact surfaces are considered smooth and non-deformable;
- the contact pressure, p_m , is considered uniformly distributed (clearances, machining and assembly errors and wear are not taken into account);
- the coefficient of friction is considered constant;
- chamfers and fillets are neglected;
- the journal and the bush are considered coaxial.

Calculation of the mean contact pressure

From the equilibrium equation of the forces in the direction of the radial force,

$$F_r = \int_{-\pi/2}^{\pi/2} dF \cos \alpha = \int_{-\pi/2}^{\pi/2} p_m B \frac{d}{2} d\alpha \cos \alpha = \frac{1}{2} p_m d B \int_{-\pi/2}^{\pi/2} \cos \alpha d\alpha =$$

$$p_m d B$$

the verification relation for the contact pressure results,

For an accepted volume of worn material V_u and an imposed durability L_h , taking into account relation (LGA-T.8.1.2), the verification relation is determined,

$$pv_m \leq (pv)_{au} \quad (\text{LGA-T.8.1.3})$$

where $(pv)_{au}$ [MPa·m/s] is the permissible pressure-velocity product from the point of view of the wear imposed over a period of time L_h ; its values depend on the materials in contact (annex LGA-P.1.1).

Thermal calculation

Model 1 (summary verification)

The heating of the bearing is determined by the specific power (per unit of contact area) lost by friction,

$$P_{sf} = \frac{P_f}{dB} = \frac{F_f v}{dB} = \frac{\mu F_r v}{dB} = \frac{\mu p_m d B v}{dB} = \mu p_m v. \quad (\text{LGA-T.8.1.4})$$

Considering the coefficient of friction μ constant, the product $p_m v$ in the relation above is an indicator of the heating level, and for verification,

$$p_m v \leq (pv)_{ai},$$

where $(pv)_{ai}$, the permissible pressure-velocity product from the point of view of the admitted heating, has values obtained experimentally for different materials and fields of use.

Model 2 (determination of the mean temperature of the bearing (oil))

The heat balance equation,

$$P_f = Q_c,$$

where P_f is the power lost by friction and Q_c the heat dissipated through the bearing housing to the environment, becomes in explicit form,

$$F_r \mu \omega \frac{d}{2} = \alpha_c A (T - T_0) \quad (\text{LGA-T.8.1.5})$$

where μ is the coefficient of friction, T – the mean temperature of the bearing, including the lubricant; T_0 – the ambient temperature; α_c – the overall heat transfer factor through the bearing housing; A – the area of the outer surface of the housing; $\omega = \pi n/30$, the angular velocity of the journal (speed n [rpm]). From this relation the mean temperature is determined,

$$p_m = \frac{F_r}{dB} \leq p_a, \text{ (LGA-T.8.1.1)}$$

where the permissible pressure, p_a , is adopted depending on the materials in contact (annex LGA-P.1.1) or from the condition that the lubricant is not squeezed out; for the design of non-standard bushes $B/D < 1.2$ is adopted (for larger values the non-uniformity of the contact pressure distribution in the axial direction increases, fig. LGA-T.8.1.1.1,b)

Wear (durability) calculation

The volume of material removed by wear,

$$V_u = k F_r l = k d B p_m v L_h \text{ (LGA-T.8.1.2)}$$

where $l = v L_h$ is the length travelled by a point on the outer surface of the journal, k – proportionality factor depending on the pair of materials and on the lubrication conditions; L_h – operating life, v – peripheral speed.

$$T = T_0 + \frac{\mu \pi n d F_r}{60 \alpha_c A} \leq T_a \text{ (LGA-T.8.1.6)}$$

where T_a is the permissible temperature.

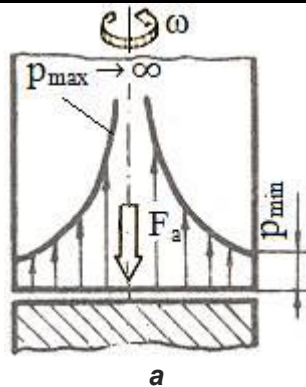
Bending calculation of the journal

In the case $B/D > 1$, maximum bending stresses can occur in the area of the stress concentrator created by the step in diameters,

$$\sigma_i = \frac{M_i}{W_z} = \frac{F_r l}{W_z} = \frac{F_r \frac{B}{2}}{\frac{\pi d^3}{32}} = \frac{16 F_r B}{\pi d^3} \leq \sigma_{ai} \text{ (LGA-T.8.1.7)}$$

Note. For shafts with increased flexibility, in relation (LGA-T.8.1.7) the force arm l is considered equal to the journal length, B .

LGA-T.8.1.2 Simplified calculation model of axial (thrust) bearings



Assumptions:

- the hyperbolic dependence of the contact pressure on the radius leads to increased pressures (theoretically ∞) in the centre of pivots with circular cross-section (fig. LGA-T.8.1.2.1,a); to avoid this disadvantage, pivots with annular cross-section are used in practice (fig. LGA-T.8.1.2.1,b);
- for the operating period, non-uniform wear of the surfaces in contact can be considered, $p v = p \omega r = \text{const.}$ ($p r = \text{const.}$);
- for the initial operating period (running-in), a constant pressure distribution $p = \text{const.}$ can be considered (fig. LGA-T.8.1.2.1,c).

Calculation of pressures under the assumption of non-uniform pressure (fig. LGA-T.8.1.2.1,b)

From the equilibrium of the forces in the axial direction,

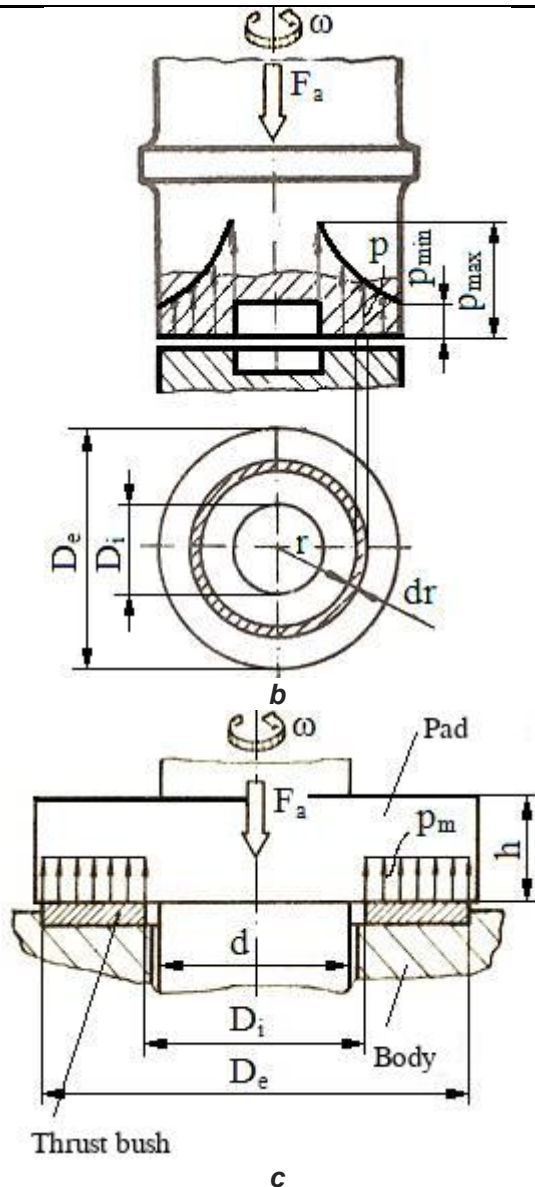


Fig. LGA-T.8.1.2.1 Simplified calculation models of axial bearings: **a** – bearing with circular contact surface and non-uniform pressure; **b** – bearing with annular contact surface and non-uniform pressure; **c** – bearing with annular contact surface and uniform pressure

$$F_a = \int_A p \, dA = \int_{D_i/2}^{D_e/2} p \, 2\pi r \, dr = 2\pi r p \int_{D_i/2}^{D_e/2} dr = \pi r p (D_e - D_i)$$

the pressure relation results

$$p = \frac{F_a}{\pi r (D_e - D_i)}, \quad (\text{LGA-T.8.1.7})$$

from which, for $r = D_e/2$ and $r = D_i/2$, one obtains,

$$p_{\min} = \frac{2F_a}{\pi D_e (D_e - D_i)} \quad \text{and, respectively,} \quad p_{\max} = \frac{2F_a}{\pi D_i (D_e - D_i)} \leq p_a$$

Calculation of pressures under the assumption of uniform pressure (fig. LGA-T.8.1.2.1,c)

$$p = \frac{F_a}{\frac{\pi}{4}(D_e^2 - D_i^2)} \leq p_a \quad (\text{LGA-T.8.1.8})$$

Thermal calculation

$$p_m v \leq (pv)_{a1},$$

The mean temperature of the bearing is determined as in the case of the radial bearing (LGA-T.8.1.6),

$$T = T_0 + \frac{\mu \pi n d F_a}{60 \alpha_c A} \leq T_a \quad (\text{LGA-T.8.1.9})$$

where $d = D_m = (D_e + D_i)/2$ is the mean diameter.

Wear (durability) calculation

$$p_m v \leq (pv)_{au} \quad (\text{LGA-T.8.1.10})$$

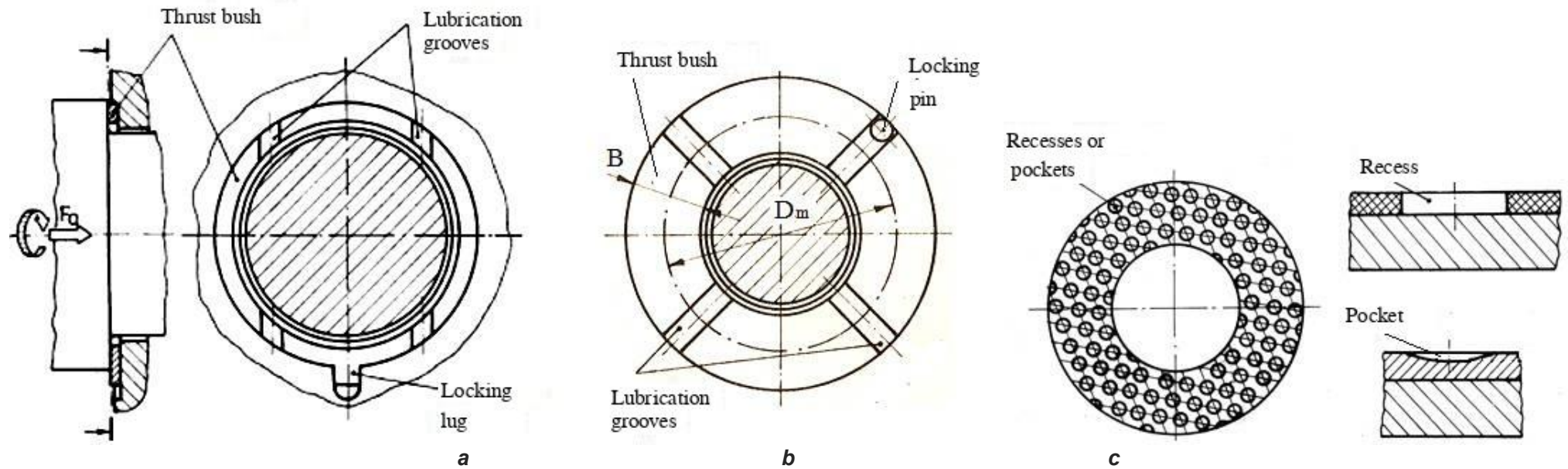
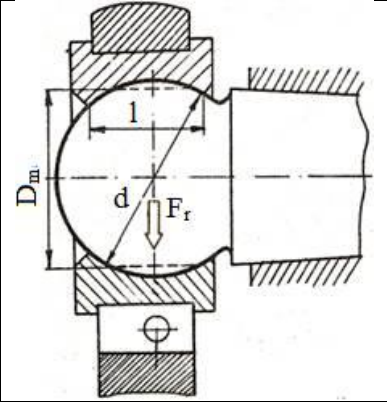
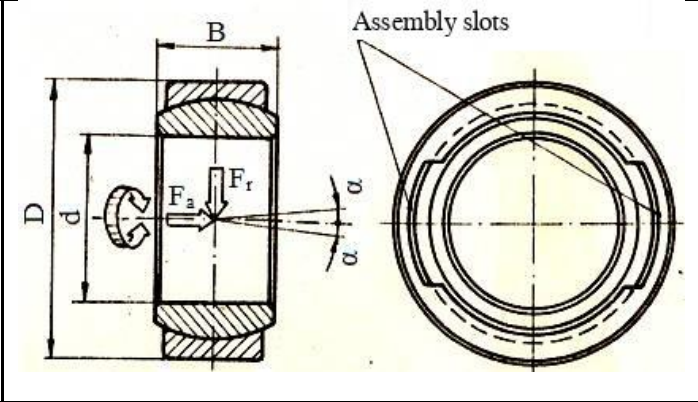


Fig. LGA-T.8.1.2.2 Constructive solutions of axial bearings: *a* – with transverse lubrication grooves; *b* – with radial lubrication grooves; *c* – with pockets

LGA-T.8.1.3 Simplified calculation models of spherical bearings

		Pressure calculation for standard joints (fig. LGA-T.8.1.3.1) Assumptions: - the spherical journal is considered cylindrical, with the mean diameter $D_m \approx 0.9 d$; - it is considered loaded only by the radial force. Mean pressure $p_m = \frac{F_r}{D_m l} = \frac{F_r}{\varphi D_m^2} \leq p_a \quad (\text{LGA-T.8.1.11})$ $\varphi = l/D_m \approx 0.7$ Durability calculation for standard joints The verification relation, $P \leq C, \quad (\text{LGA-T.8.1.12})$ where P is the equivalent dynamic load, $P = F_r + YF_a$, with Y equal to 6 for static loads and 12 for dynamic loads, and C is the dynamic load rating (from the catalogue), defined as the radial load that the spherical journal can carry without being damaged by scuffing when it oscillates through $\pm 30^\circ$, for 5000 cycles at a contact pressure of 250 MPa (C_{5000}) or for 10000 cycles at a contact pressure of 20 MPa (C_{5000}).
a	b	
Fig. LGA-T.8.1.3.1 Simplified calculation models of spherical bearings (joints): a – with non-standard structure; b – with standard structure		

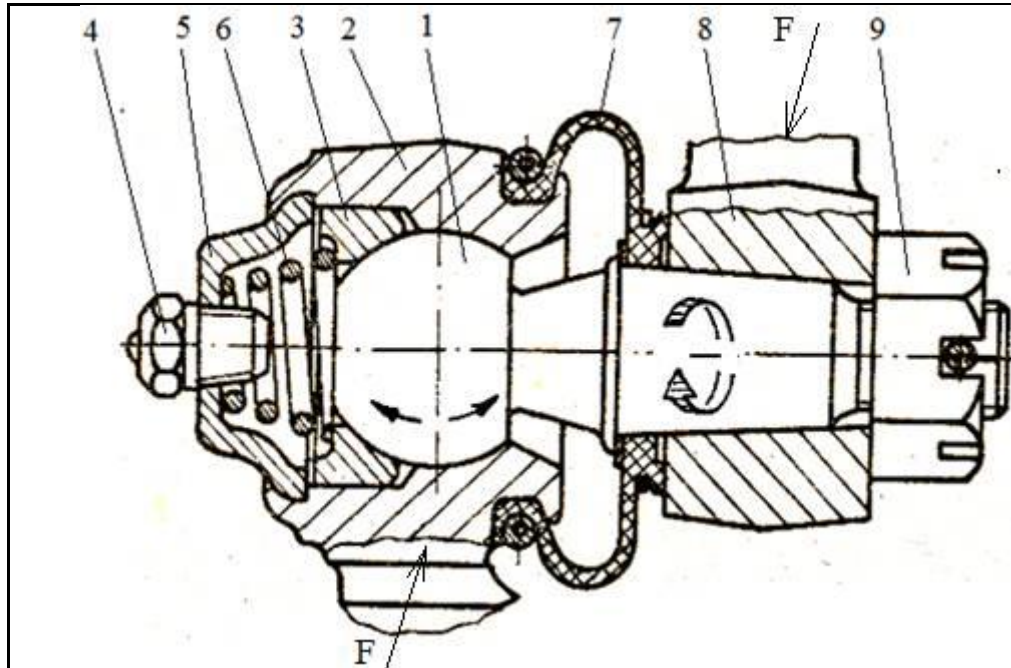


Fig. LGA-T.8.1.3.2 Spherical joint of the steering-suspension mechanisms of motor vehicles

Construction and operation:

The force is transmitted from arm 8, by friction in the conical press-fit assembly, to the spherical journal 1 and then, by shape, to the spherical bearing made of two hemispheres machined in body 2 and in half-bush 3, to the other arm, which is integral with body 2. The conical press-fit assembly is tightened with the castellated nut 9, which is secured with a split pin (cotter pin). Permanent contact in the spherical joint is ensured by the conical helical spring 6, supported on the support washer 5. The spherical joint is lubricated with grease through the ball lubricator 4. The joint is sealed with the boot (bellows) 7.

LGA-T.8.2 CALCULATION MODELS OF BEARINGS WITH FLUID FRICTION

LGA-T.8.2.1 Calculation models of hydrodynamically lubricated bearings

LGA-T.8.2.1.1 Calculation model of hydrodynamic radial bearings

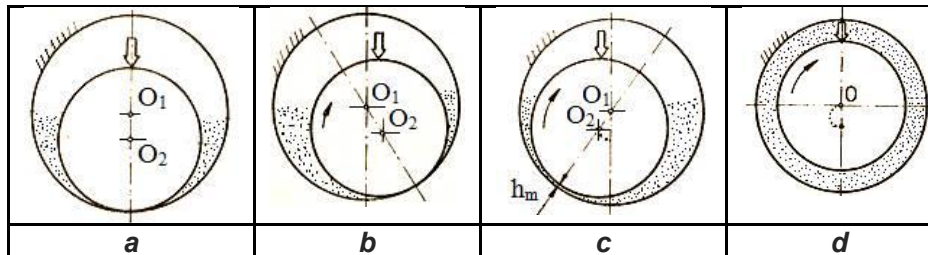


Fig. LGA-T.8.2.1 Formation of the lubricant film: **a** – rest; **b** – start-up and low speed; **c** – development of the lubricant film; **d** – very high speeds (theoretical case)

Formation of the load-carrying lubricant film:

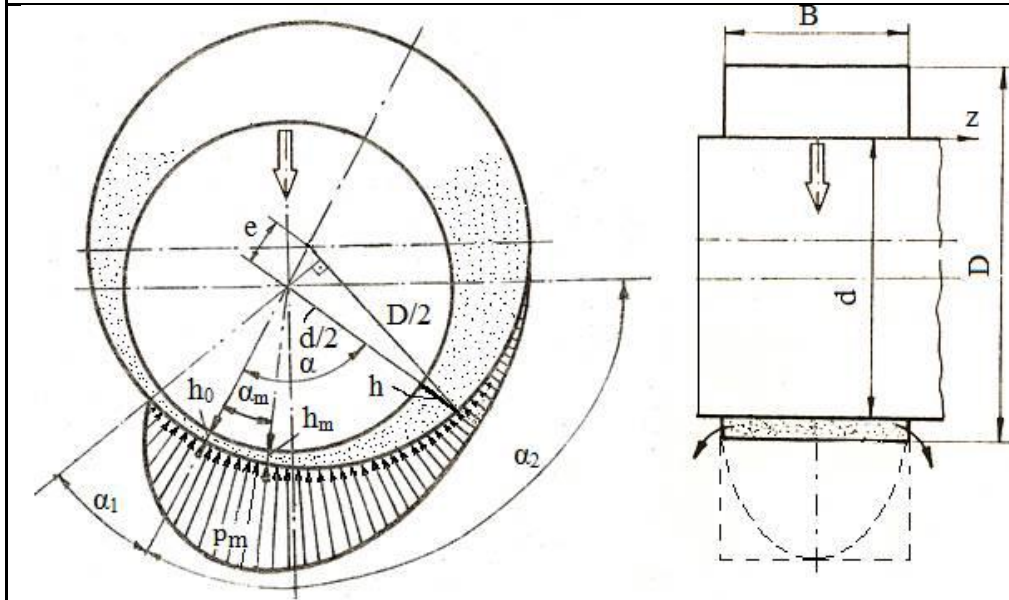
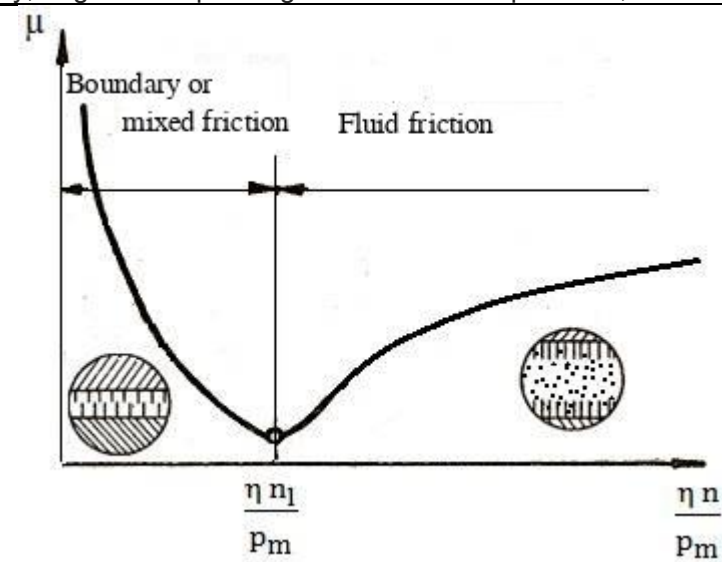
a. the journal is at rest; direct metal-to-metal contact occurs between the surfaces in the lower zone (fig. LGA-T.8.2.1,a);
b. start-up of the journal; direct contact and dry or semi-dry friction at low speed (fig. LGA-T.8.2.1,b);
c. the speed increases and, starting from a limit value, the lubricant film develops; as a result of the wedge effect conditions, the lubricant is entrained in the direction of motion (fig. LGA-T.8.2.1,c) and there is a gradual transition from dry friction to mixed friction and finally to fluid friction (fig. LGA-T.8.2.2,b);
d. at extremely high speeds the relative position of the surfaces in contact continues to change (the thickness of the lubricant film increases); theoretically, for $n \rightarrow \infty$, the journal and the bush become concentric.

Calculation assumptions:

- the conditions of hydrodynamic lubrication exist: the wedge geometry due to the journal-bush clearance; lubricant is present in the contact zone and there is a relative rotational speed between journal and bush;
- the journal is considered very long (unidirectional fluid flow);
- the effects of centrifugal forces are neglected

Calculation parameters:

d – journal diameter; D – bush diameter; B – bush width; h – lubricant film thickness in an arbitrary section; e – eccentricity; $j = (D-d)/2$, diametral clearance; $\Psi = (D-d)/d = j/d$, relative clearance; $\varepsilon = e/(j/2)$, relative eccentricity; $\delta = h_0/(j/2)$, relative minimum thickness ($\delta + \varepsilon = 1$); h_0 , h , h_m – lubricant film thicknesses (minimum, current and, respectively, corresponding to the maximum pressure, p_m); α , α_1 , α_2 , α_m – current, inlet, outlet angle and, respectively, angle corresponding to the maximum pressure; F_r – radial force

**a****b****Fig. LGA-T.8.2.2 Calculation model of hydrodynamic radial bearings: a – calculation scheme; b – associated Stribeck curve****Calculation model of the hydrodynamic lubrication parameters**

Particularization of the Reynolds equation (see rel. ETB-T.4.3.1.1) for hydrodynamic radial bearings, considering $x = d/2 \alpha$, $dx = d/2 d\alpha$, $v = \omega d/2$ and the variation of the lubricant film thickness (fig. LGA-T.8.2.2,a),

$$h \approx \frac{D}{2} - \frac{d}{2} - e \cos \alpha = e \left(\frac{D-d}{2e} - \cos \alpha \right) = e (\varepsilon - \cos \alpha)$$

becomes,

$$\frac{1}{d/2} \frac{\partial}{\partial \alpha} \left(e^3 (\varepsilon - \cos \alpha)^3 \frac{\partial p}{\partial \alpha} \right) = -6 \eta \omega \frac{\partial}{\partial \alpha} (e(\varepsilon - \cos \alpha))$$

(LGA-T.8.2.1)

By integrating this equation twice with respect to the angular variable α , the pressure variation inside the lubricant film is obtained, $p = p(\alpha, d/2, \omega, \varepsilon, \eta)$.

The load-carrying capacity of the lubricant film is obtained by integration, as the sum of the pressures distributed over the contact surface,

Friction torque

$$M_f = F_f \frac{d}{2} = \mu_f F_r \frac{d}{2} = c_f \Psi F_r \frac{d}{2}, \quad (\text{LGA-T.8.2.6})$$

where $c_f = \mu_a/\Psi = 2 \pi^2 S$ is the friction loss coefficient [Olaru, 2002]; the sliding friction coefficient, $\mu_f = c_f \Psi = 2 \pi^2 S \Psi = 2 \pi^2 \eta n p_m \Psi$.

Thermal calculation model of the hydrodynamically lubricated bearing

The heat balance equation,

$$P_f = W_1 + W_2 \quad (\text{LGA-T.8.2.7})$$

expresses the equality between the power lost by friction, P_f , and the sum of the heats W_1 and W_2 removed and transferred to the lubricant (in circulation) and, respectively, to the environment (atmosphere).

After explicitation, relation (LGA-T.8.2.7) becomes,

$$F_r = \int_{\alpha_1}^{\alpha_2} p \sin(\alpha_1 - \alpha) \frac{d}{2} B d\alpha = \dots = \frac{\eta n}{\Psi^2} d B S \quad (\text{LGA-T.8.2.2})$$

where S_0 is the Sommerfeld number or the load-carrying number, a dimensionless quantity. In design, the load-carrying coefficient, $C_p = 1/S_0$, is frequently used.

Taking into account relation (LGA-T.8.2.2), the conventional mean pressure results,

$$p_m = \frac{F_r}{d B} = \frac{\eta n}{\Psi^2} S \quad (\text{LGA-T.8.2.3})$$

When the mean pressure is known (from experimental determinations), from relation (LGA-T.8.2.3) one determines,

$$S = \frac{p_m \Psi^2}{\eta n} \quad (\text{LGA-T.8.2.4})$$

where η is the dynamic viscosity and n – the speed in rpm.

Fig. LGA-T.8.2.3 shows the dependence of the Sommerfeld number on the relative minimum thickness, $\delta = h_0/(j/2)$, on the relative eccentricity, $\varepsilon = e/(j/2)$, and on the ratio B/d . In design, knowing the value of the Sommerfeld number, the minimum thickness of the lubricant film, h_0 , can be determined; it must be greater than the sum of the roughnesses of the journal and the bush.

Fig. LGA-T.8.2.3 Variation of the relative minimum thickness, $\delta = h_0/(j/2)$, as a function of the Sommerfeld number, $S = \eta n/p_m \Psi^2$, and of the ratio B/d [Olaru, 2002]

$$F_r \mu_f \omega \frac{d}{2} = c q (T - T_i) + \alpha_c A (T - T_0), \quad (\text{LGA-T.8.2.8})$$

where c is the specific heat of the lubricant, q – the lubricant flow rate, T – the mean temperature of the bearing, including the lubricant, T_i – the oil inlet temperature ($T = T_i + 5 \dots 8$ °C), T_0 – the ambient temperature, α_c – the overall heat transfer factor through the bearing housing, A – the area of the outer surface of the housing; $\omega = \pi n/30$, the angular velocity of the journal (speed n [rpm]). In design, relation (LGA-T.8.2.8) is used to determine the operating temperature T , which must be lower than the permissible operating temperature of the oil, $T_a = 60 \dots 80$ °C

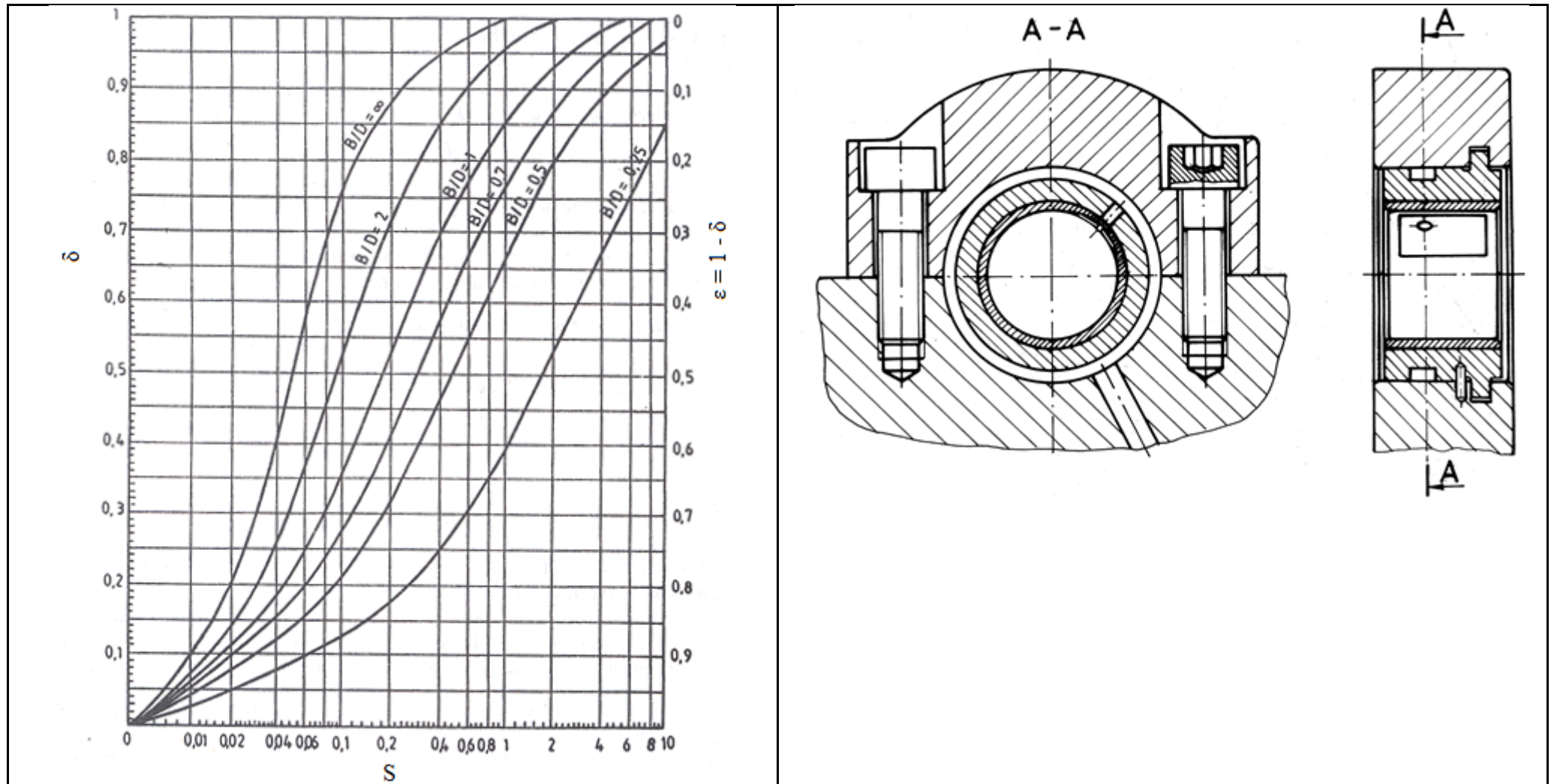
Note.

a. The radial clearance influences the operation, friction and heating of the bearing; the clearance changes with the difference between the operating and assembly temperatures; its value changes during operation, especially rapidly during running-in; large clearances can lead to increased angular deformations of the journals; relative clearance, $\Psi = (0.3 \dots 3) \text{‰}$ for bearings with metallic bushes; recommended fits for the journal-bush subassembly can be: H7/g6 or H7/f7, for journals loaded at high pressures and operating at low speeds; H7/d8 or H7/e8 for journals operating at high speeds

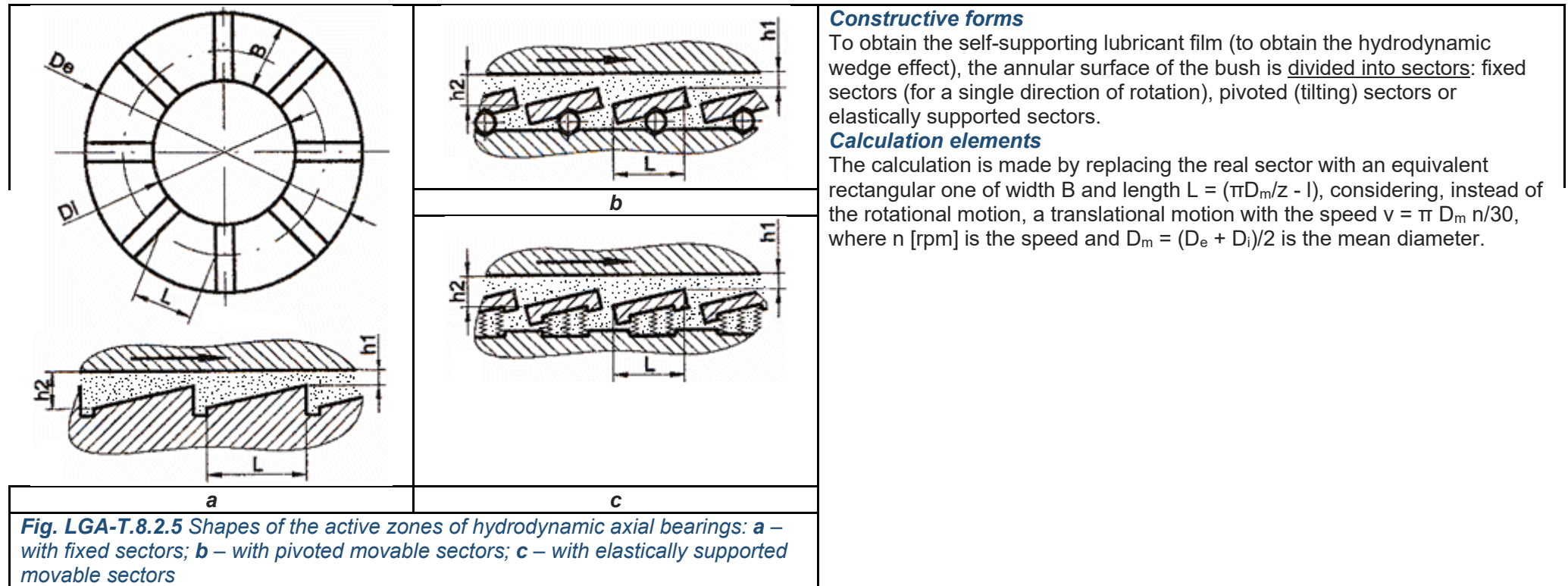
b. The minimum thickness of the lubricant film, h_0 , must be greater than the lower permissible limit determined by the manufacturing conditions (roughness) of the surfaces and by the elastic deformation of the journal; for normal, carefully machined bearings the condition for the existence of fluid friction can be expressed as $h_0 > 10 \text{ μm}$; the minimum permissible thickness of the lubricant film, especially in the case of lubrication through an external circuit, must be greater by the filtration level, $10 \dots 15 \text{ μm}$.

c. From the point of view of lubrication and cooling, hydrodynamic bearings can have their own lubrication system and cooling through the bearing body, or lubrication through an external circuit and cooling by the lubricant.

Fig. LGA-T.8.2.4 Hydrodynamic radial bearing with a solid bush fed through a pocket

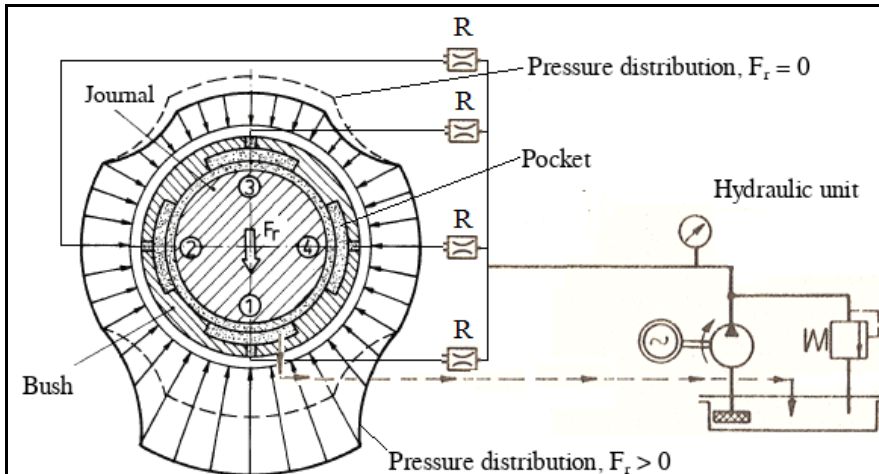


LGA-T.8.2.1.2 Calculation model of hydrodynamic axial bearings



LGA-T.8.2.2 Calculation models of hydrostatically lubricated bearings

LGA-T.8.2.2.1 Calculation model of hydrostatic radial bearings



Functional and constructive particularities:

- They are used for high loads and low speeds (when the self-supporting hydrodynamic film cannot be achieved) and/or when increased precision is required (when hydrodynamic bearings with imposed clearance are unsuitable); they have low friction, even at start-up.
- As a result of the lubricant (oil) being introduced under pressure by the hydraulic unit (supply installation), through the hydraulic restrictors R, into the pockets (recesses) machined in the bush, a load-carrying lubricant film develops even in the absence of rotational motion.
- To carry radial loads, partial (open) radial bearings can be used, which can operate stably with a single pocket (unidirectional radial force), or complete radial bearings, which can operate stably only with at least three pockets (usually 4 or 6 pockets, fig. LGA-T.8.2.6).

Calculation particularities:

They are calculated similarly to hydrodynamic bearings:

- the load-carrying force,
- the lubricant flow rate,
- the power consumed by friction and pumping

Fig. LGA-T.8.2.6 Functional scheme of hydrostatic radial bearings

LGA-T.8.2.2.2 Calculation model of hydrostatic axial bearings

TCR. BELT DRIVES

TCR-T. CONSTRUCTION AND MODELING OF BELT DRIVES

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TCR-T.1 DEFINITION

Belt drives are mechanical transmissions whose role is to transmit energy from a driving shaft to one or more driven shafts by means of an endless flexible traction element, called a belt.

The transmission belt is an endless flexible element, made of non-metallic materials, which transmits the rotational motion and the load from a driving pulley to one or more driven pulleys.

The belt pulley is the metallic element which transmits the load (by shape or by friction) from the input element to the belt (driving pulley) or, respectively, from the belt to the output element (driven pulley).

TCR-T.2 CONSTRUCTIVE STRUCTURE

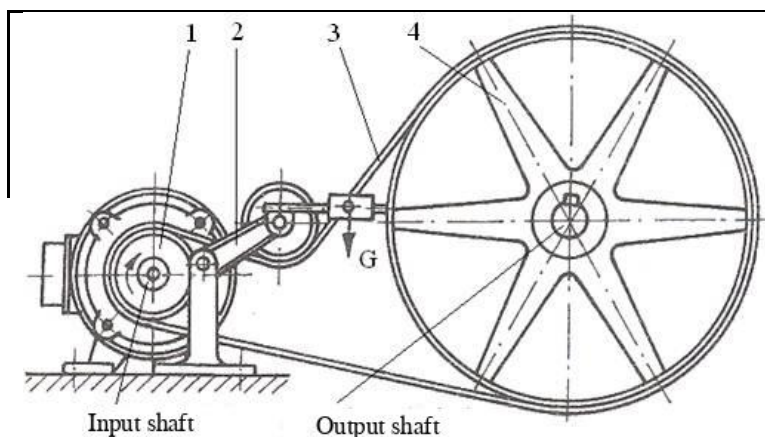


Fig. TCR-T.2.1 Structure of a belt drive [Gafițanu, 2002]

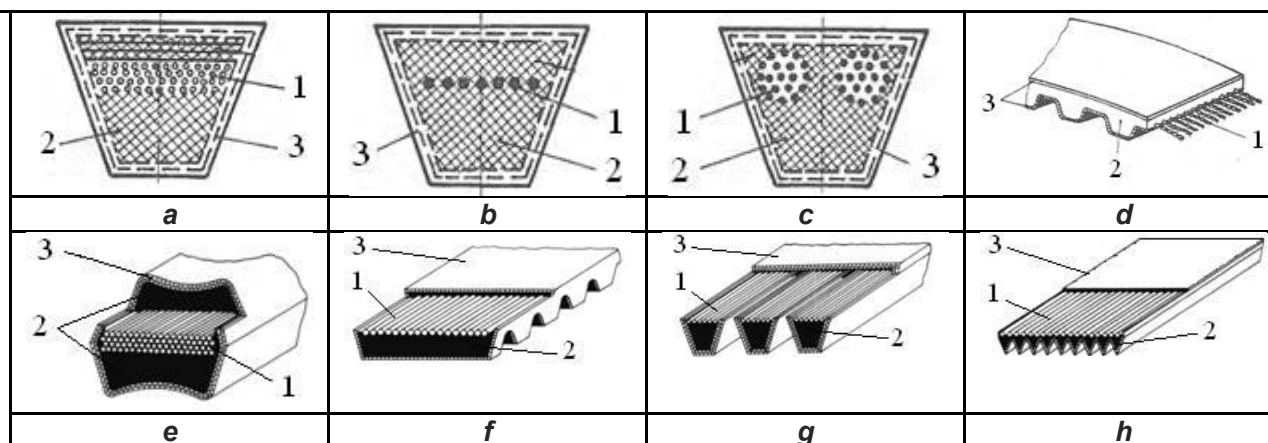


Fig. TCR-T.2.2 Types and structures of belts: **a** – V-belt with fabric tension member; **b** – V-belt with textile cord tension member; **c** – V-belt with cable-type tension member; **d** – toothed (synchronous) belt; **e** – double V-belt (hexagonal); **f** – wide toothed V-belt; **g** – multiple (banded) V-belt; **h** – multi-ribbed belt (poly-V)

Note. Meanings of the components in fig. TCR-T.2.1: 1 – driving pulley; 2 – belt tensioning system (mechanism actuated by the weight G); 3 – belt; 4 – driven pulley.

Meanings of the parts in fig. TCR-T.2.2: 1 – tension member (made of textile, metallic or plastic materials); 2 – supporting structure (usually made of rubber); 3 – protective cover in one or more layers (usually made of rubberized fabric).

TCR-T.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Main areas of use: transmissions of motor vehicles, agricultural machines, tractors, technological equipment, machine tools, etc. In general, they are used in almost all transmissions in which the overall-dimension conditions require large distances between the pulleys.

Tab. TCR-T.3.1 *Advantages and disadvantages of belt drives [Rădulescu, 1985]*

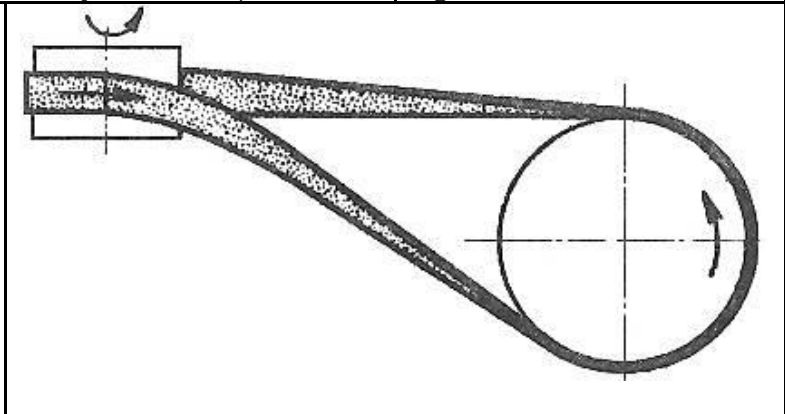
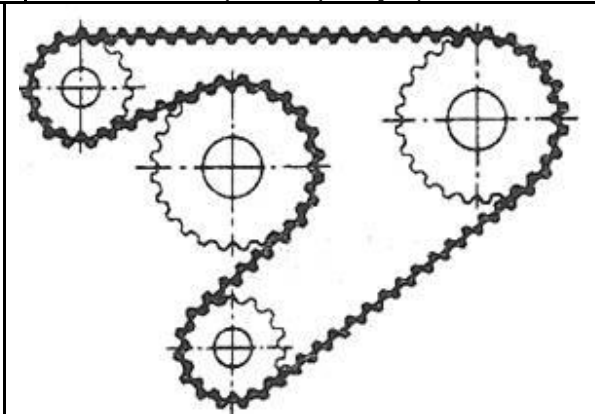
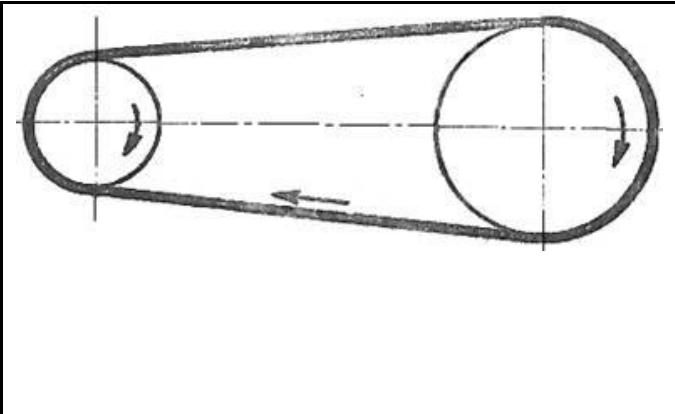
Advantages*	Disadvantages*
- rotational motion and power can be transmitted over a distance; - smooth, noiseless operation; - damping of shocks and vibrations; - protection of some components against overloads; - low manufacturing and assembly accuracy is required, so they are relatively inexpensive; - several shafts can be driven simultaneously	- large overall dimensions compared to gear transmissions; - limited load-carrying capacity; - elastic slip (in the case of friction drives); - they induce high loads in shafts and bearings, because a relatively high initial tensioning force is required; - limited durability (ageing of the rubber of the supporting structure and wear of the protective cover); increased sensitivity to heat and humidity; - they produce electrostatic charges
* Compared to other types of transmissions (gear drives, chain drives, etc.)	

TCR-T.4 CLASSIFICATION

Tab. TCR-T.4.1 *Classification of belt drives [Jula, 1986; Butnariu, 2009]*

Criterion	Type	Scheme
Way of transmitting the load from the pulley to the belt (or from the belt to the pulley)	By friction (asynchronous) – the case of V-belts, flat, round and ribbed belts	Fig. TCR-T.4.1,a,c
	By shape (synchronous) – the case of toothed belts	Fig. TCR-T.4.1,b
Number of driven shafts	With one driven shaft	Fig. TCR-T.4.1,a,c
	With several driven shafts	Fig. TCR-T.4.1,b
Type of belt tensioning system	With permanent tensioning, without tensioning pulley	Fig. TCR-T.9.1.1,a
	With permanent tensioning, with (adjustable) tensioning pulley	Fig. TCR-T.9.1.1,b,c
	With automatic tensioning	Fig. TCR-T.9.2.1
Shape of the belt cross-section	With flat belts	Fig. TCR-T.4.1,c
	With V-belts	Fig. TCR-T.2.2,a,b,c,e,f,g
	With multi-ribbed (poly-V) belts	Fig. TCR-T.2.2,h
Position of the pulley axes	With parallel axes	Fig. TCR-T.4.1,a,b
	With crossed axes	Fig. TCR-T.4.1,c
Belt material	Single material (leather, textile strips)	-
	Composite (with compound structures: tension member, supporting material, cover)	Fig. TCR-T.2.2

Number of belts	With one belt	Fig. TCR-T.4.1; 4.2,e
	With two or more belts	Fig. TCR-T.4.2,a,b,c
Shape of the belt pulley	With one-piece pulleys (forged, cast)	Fig. TCR-T.4.2,a
	With multi-component pulleys (welded, mechanically assembled)	Fig. TCR-T.4.2,b,c,d



a

b

c

Fig. TCR-T.4.1 Types of belt drives in relation to the positions of the pulley axes: **a** – with parallel axes and one driven pulley; **b** – with parallel axes and three driven pulleys (toothed belt); **c** – with crossed axes and one driven pulley

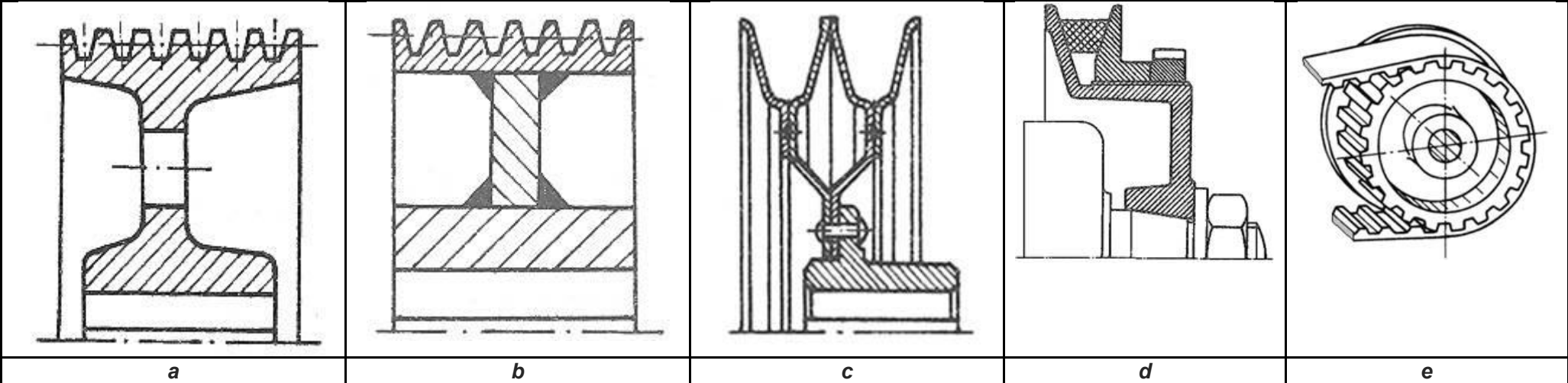


Fig. TCR-T.4.2 Types of belt pulleys: **a** – forged or cast; **b** – welded; **c** – assembled, non-dismountable; **d** – assembled, dismountable (adjustable); **e** – toothed

TCR-T.5 MATERIALS AND TECHNOLOGIES

Belts are elements with a complex structure, manufactured by specialized companies according to technologies that involve dedicated equipment.

Belt pulleys are usually metallic structures: one-piece (fig. TCR-T.4.2,a), made of non-alloy carbon structural steels (S235, S275, S355) or non-alloy cast steels; multi-component welded (fig. TCR-T.4.2,b), made of weldable steels (S235, S275, S355); multi-component assembled (fig. TCR-T.4.2,c,d), with components made of non-alloy carbon structural steels or non-alloy quenched and tempered steels (C35, C45, C55). Toothed belt pulleys can also be made of aluminium alloys or sintered materials.

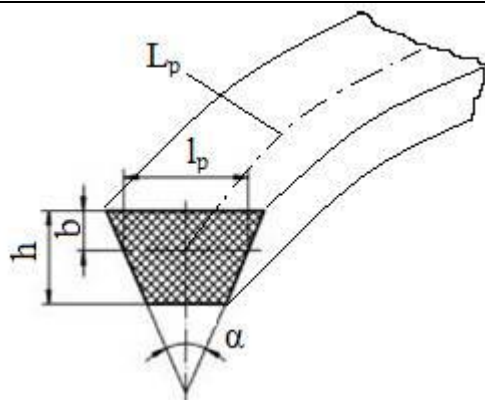
TCR-T.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. TCR-T.6.1 Forms and causes of improper behavior or removal from service

Forms	Consequences	Occurrence	Causes	Manifestation	Avoidance
Damage of the active surfaces of the belts	Improper behavior (with irregularities, vibrations and noise)	Mainly in friction drives	Wear of the cover	Non-uniform motion with vibrations; increased slip	Proper selection and mounting; optimal tensioning of the belt
Belt breakage	Removal from service	Drives overloaded by static loads Too large centre distances lead to vibrations of the belt spans	Exceeding the static or fatigue strength of the material of the supporting structure (rubber)	Interruption of the motion transmission	Limiting the maximum static or fatigue stresses to permissible values, limiting the operating overloads and/or the bending frequency of the belt

TCR-T.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

TCR-T.7.1 NARROW V-BELTS

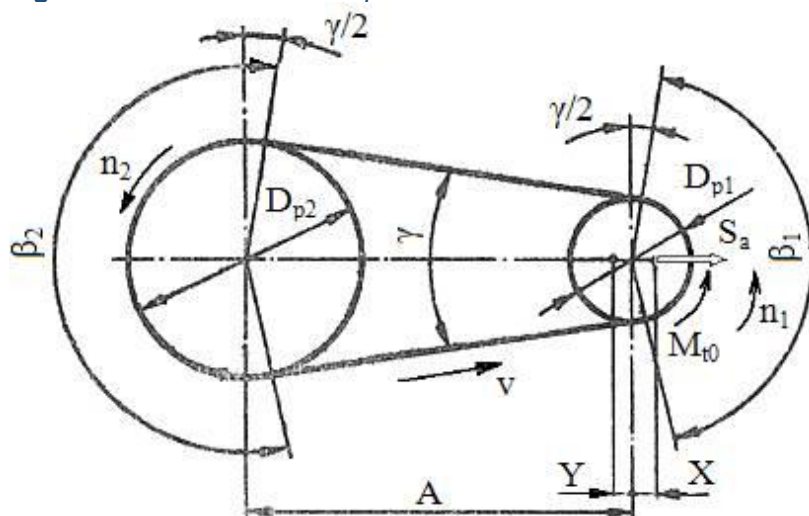


Main geometric parameters of the V-belt (fig. TCR-T.7.1.1):

- **Belt width**, l_p – the pitch width of the belt at the level of the tension fibres (which are neither compressed nor stretched during the operation of the belt).
- **Belt height**, h .
- **Belt angle** (e.g. $\alpha = 40^\circ$, for the classical V-belt).
- **Belt length**, L_p (pitch length, at the level of the tension fibres).
- **Number of grooves/belts**, z .

Main geometric parameters of the V-belt drive (fig. TCR-T.7.1.2):

- **Pitch diameter of the pulley** (driving/driven), $D_{p1,2}$ – the virtual diameter of the pulley corresponding to the tension fibre of the belt wrapped on the pulley.
- **Centre distance**, A .
- **Wrap angle of the belt on the pulley** (driving/driven), $\beta_{1,2}$.

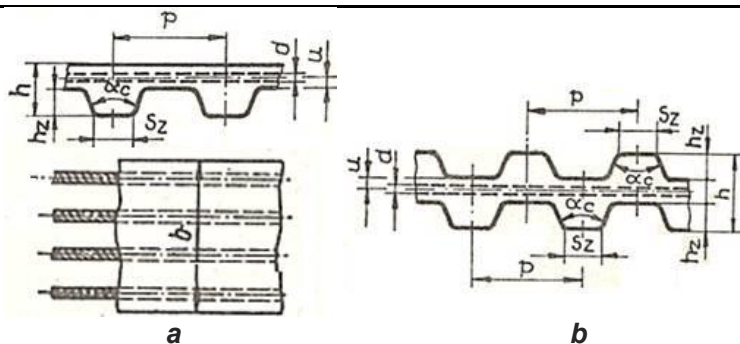
Fig. TCR-T.7.1.1 Geometric parameters of the V-belt**Fig. TCR-T.7.1.2** Geometric parameters of the V-belt drive without tensioning pulley [STAS 1163]

- Angle between the belt spans, γ .
- Centre distance adjustment travel, X , for tensioning the belt.
- Centre distance adjustment travel, Y , required for mounting the belt.

Main kineto-static parameters of the V-belt drive (fig. TCR-T.7.1.2):

- Pulley speed (driving/driven), $n_{1,2}$.
- Nominal transmitted power, P .
- Nominal power transmitted by one belt, P_0 .
- Peripheral speed of the belt, v .
- Belt tensioning force, S_a .
- Transmission ratio, $i = D_{p2}/D_{p1} = n_1/n_2$.
- Torque at the driving pulley (for one belt), M_{t0} .
- Bending frequency of the belt, f .

TCR-T.7.2 TOOTHED BELTS

**Fig. TCR-T.7.2.1** Geometric parameters of wide toothed belts with trapezoidal profile: **a** – with teeth on one side; **b** – with teeth on both sides

Main geometric parameters of wide toothed belts (fig. TCR-T.7.2.1):

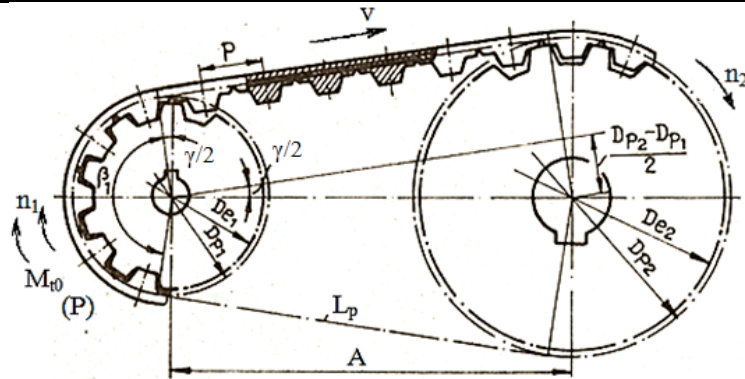
- Belt width, b .
- Belt height, h .
- Tooth profile angle, $\alpha_c = 40^\circ \dots 50^\circ$.
- Belt length, L_p (pitch length, at the level of the tension fibres).
- Belt pitch, p .
- Number of belt teeth, $z_c = L_p/p$.
- Tooth thickness, s_z .
- Tooth height, h_z .

Main geometric parameters of a wide toothed belt drive (fig. TCR-T.7.2.2):

- Pitch diameter of the pulley (driving/driven), $D_{p1,2}$ – the virtual diameter of the pulley corresponding to the tension fibre of the belt wrapped on the pulley.
- Outside diameter of the pulley (driving/driven), $D_{e1,2}$.
- Centre distance, A .
- Wrap angle of the belt on the pulley (driving/driven), $\beta_{1,2}$.
- Angle between the belt spans, γ .

Main kineto-static parameters of a wide toothed belt drive (fig. TCR-T.7.2.2):

- Pulley speed (driving/driven), $n_{1,2}$.
- Nominal transmitted power, P .
- Peripheral speed of the belt, v .



- Transmission ratio, $i = D_{p2}/D_{p1} = n_1/n_2$
- Torque at the driving pulley, M_{t0}

Fig. TCR-T.7.2.2 Geometric parameters of the wide toothed belt drive

To be continued...

TLA. CHAIN DRIVES

TLA-T. CONSTRUCTION AND MODELING OF CHAIN DRIVES

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TLA-T.1 DEFINITION

Chain drives transmit the torque from a driving shaft to one or more driven shafts by means of an articulated metallic traction subassembly, called a chain, which meshes with sprockets mounted on the shafts.

The chain is a flexible (articulated) subassembly made of metallic materials, which transmits the rotational motion and the torque, by shape, from a driving sprocket to one or more driven sprockets.

The sprocket (driving or driven) is the toothed metallic element which transmits the load, by shape, from an input element to the chain or, respectively, from the chain to the output element.

TLA-T.2 CONSTRUCTIVE STRUCTURES

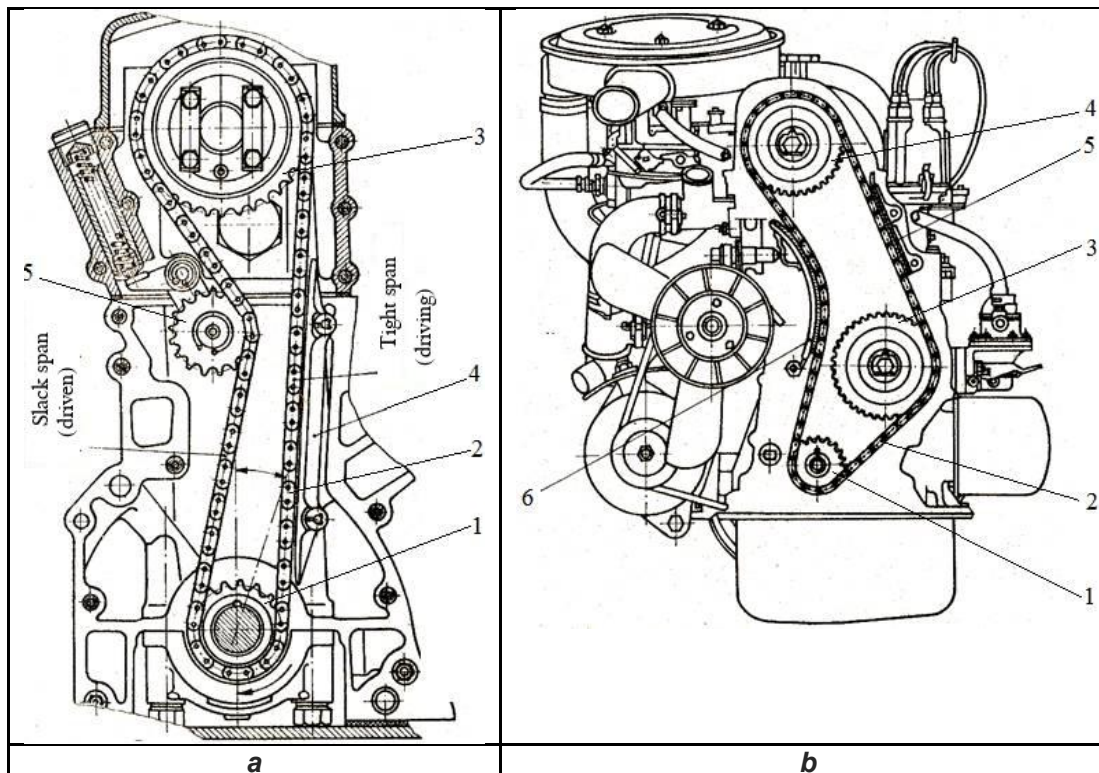


Fig. TLA-T.2.1 Chain drives for the timing (valve-train) mechanisms of internal combustion engines: **a** – with one driven sprocket; **b** – with two driven sprockets

Operation and construction:

Variant in fig. TLA-T.2.1,a

The torque is transmitted from the crankshaft to the driving sprocket 1 and, by shape, to the chain 2 and to the driven sprocket 3, mounted on the camshaft. The chain is tensioned by sprocket 5, pressed with a constant force by a helical spring system. The oscillations (vibrations) of the driving span are reduced by means of the guide shoe 4.

Variant in fig. TLA-T.2.1,b

The torque is transmitted from the crankshaft to the driving sprocket 1 and, by shape, to the chain 2 and to the driven sprockets 3 and 4. The chain is tensioned by shoe 6. The oscillations (vibrations) of the driving span are damped by means of the damping shoe 5.

TLA-T.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Main areas of use: agricultural machines, tractors, technological equipment, transport vehicles (mopeds, motorcycles, bicycles, etc.), internal combustion engines.

Tab. TLA-T.3.1 Advantages and disadvantages of chain drives

Advantages*	Disadvantages*
- they can be used over a wide range of centre distances; - high torques can be transmitted; - constant mean transmission ratios (no relative slip); - high efficiency ($\eta = 0.95 \dots 0.98$); - lower shaft loads than in the case of belt drives; - operation under various operating conditions (humidity, high temperatures)	- non-uniform motion of the driven sprocket(s), due to the polygonal wrapping of the chain on the sprocket, which produces additional dynamic loads, vibrations and noise in operation; - tensioning systems are needed to ensure the meshing of the chain with the sprocket, to reduce vibrations and to compensate for the wear of the joints; - higher manufacturing and assembly accuracy and more demanding maintenance than in the case of belt drives
* Compared to other types of transmissions (gear drives, belt drives, etc.)	

TLA-T.4 CLASSIFICATION

Tab. TLA-T.4.1 Classification of chain drives

Criterion	Type	Scheme	Notes
Purpose of use	Hoisting	Fig. TLA-T.4.2,a	The chain has welded links; it operates at low speeds ($v < 0.25$ m/s); low manufacturing and assembly accuracy
	Conveying	Fig. TLA-T.4.2,b	The chains (with link plates, scrapers, hooks, etc.) are parts of mechanical conveyors with speeds $v < 4$ m/s
	Power transmission with constant transmission ratio	Fig. TLA-T.4.2,c	The torque is transmitted by shape from the driving sprocket to the chain and from the chain, also by shape, to the driven sprocket
	Power transmission with variable transmission ratio	Fig. TLA-T.4.2,d	The torque is transmitted by shape or by friction from the driving wheel to the chain and from the chain, also by shape or by friction, to the driven wheel
Number of driven shafts	With one driven shaft	Fig. TLA-T.2.1,a	
	With two or more driven shafts	Fig. TLA-T.2.1,b	
Type of chain tensioning system	With tensioning sprocket	Fig. TLA-T.2.1,a	With mechanical (manual, adjustable spring) or hydraulic actuation
	With tensioning shoe	Fig. TLA-T.2.1,b	
Type of chain	Pin chains with single or multiple link plates	Fig. TLA-T.4.3,a,b	Transmission chains with pin – inner link plate joints; the sprocket-chain meshing takes place by sliding; used for small loads and low speeds ($v < 0.3$ m/s)

	Bush chains	Fig. TLA-T.4.3,c	Chains with pin-bush joints; the sprocket-chain meshing takes place by sliding; low speeds ($v < 3$ m/s)
	Roller chains	Fig. TLA-T.4.3,d	The sprocket-chain meshing takes place by rolling
	Toothed (silent) chains	Fig. TLA-T.4.3,e	The torque is transmitted by the conformal contact between the toothed link plate and the sprocket flank
Number of strands	With one strand (simplex)	Fig. TLA-T.4.3,d	Transmission of small torques
	With two or more strands	Fig. TLA-T.4.4	Transmission of higher torques

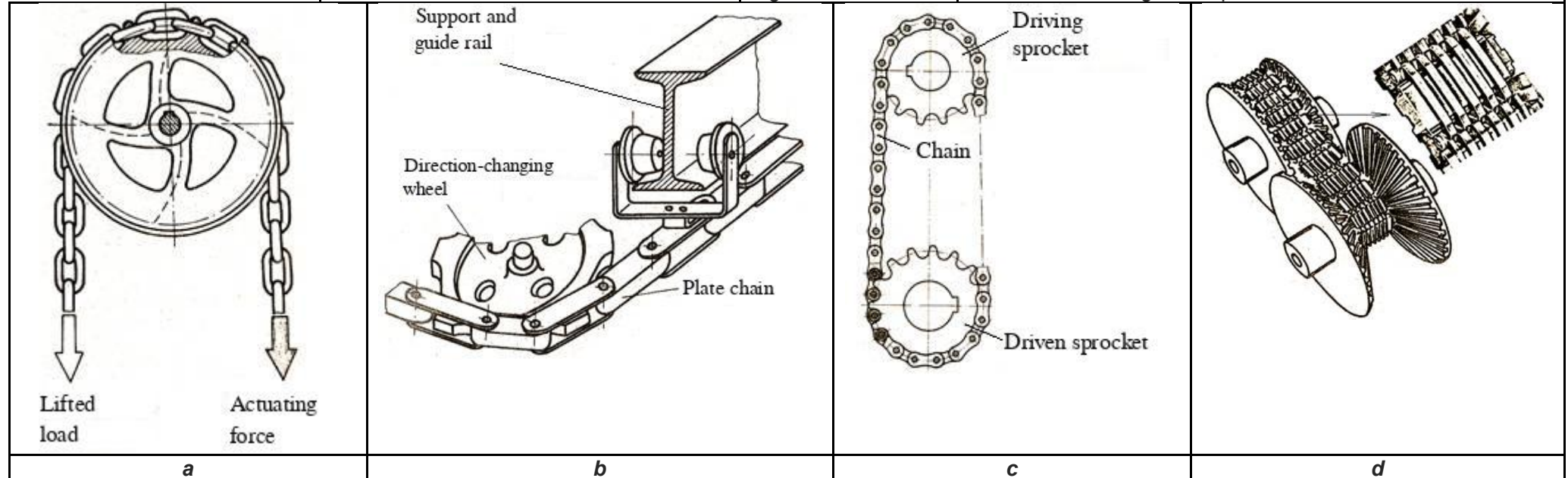


Fig. TLA-T.4.2 Types of chain drives: **a** – hoisting; **b** – conveying; **c** – for torque transmission with constant transmission ratio; **d** – for torque transmission by shape with variable transmission ratio

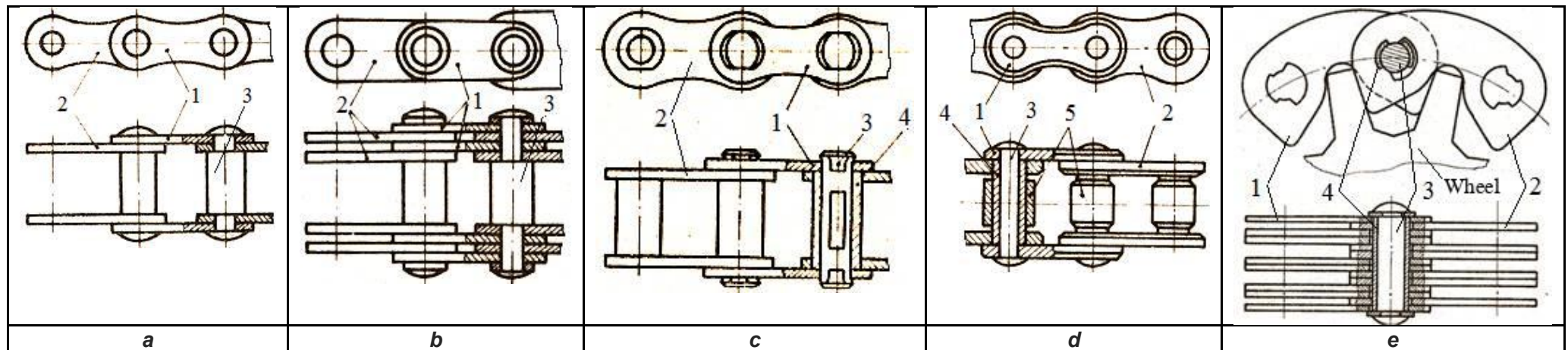


Fig. TLA-T.4.3 Types and structures of chains (1 – outer link plate, 2 – inner link plate, 3 – pin, 4 – bush, 5 – roller): **a** – with pins (Gall type) and single link plates; **b** – with pins (Gall type) and multiple link plates; **c** – with pins, link plates and bushes; **d** – with pins, link plates, bushes and rollers; **e** – with toothed link plates, pins and segmented bushes

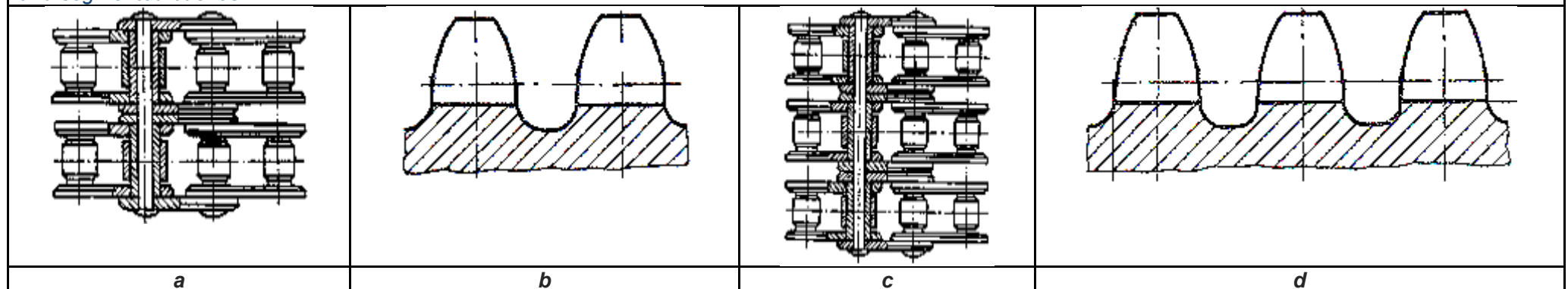


Fig. TLA-T.4.4 Structures of chain drives with several strands: **a** – chain with two strands (duplex); **b** – sprocket with two rows of teeth; **c** – chain with three strands (triplex); **d** – sprocket with three rows of teeth

TLA-T.5 MATERIALS AND TECHNOLOGIES

Chains are subassemblies with a complex structure, manufactured by specialized companies according to technologies that involve dedicated equipment; the link plates are made of cold-rolled flat strip of non-alloy (C45, C50) or alloy (40Cr10, 35CrNi15) quenched and tempered steels, hardened and tempered to 270...350 HB; the joint elements (pins, bushes and rollers) are recommended to be made of case-hardening steels (C15, C20, 13CrNi35, etc.), carburized and hardened to 45...60 HRC.

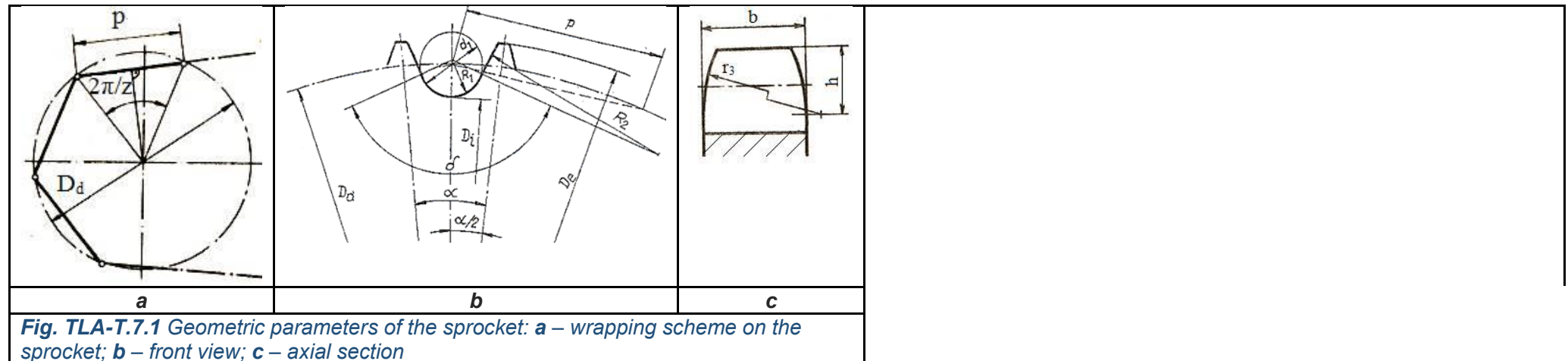
Sprockets are usually metallic structures which, for lightly loaded drives, are made of non-alloy quenched and tempered carbon steels (C35, C45, C50) or, for high loads and speeds, of case-hardening steels (C15, C20, 13CrNi35, 15Cr07, 17MoCr10, etc.) with a hardness after treatment of 48...58 HRC.

TLA-T.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. TLA-T.6.1 Forms and causes of improper behavior or removal from service

Forms	Consequences	Occurrence	Causes	Manifestation	Avoidance
Damage of the active surfaces of the chain and sprockets	Improper behavior (with irregularities, vibrations and noise)	In all properly calculated and manufactured chain drives	Wear of the joint elements (pins, bushes or rollers) and/or of the active surfaces of the sprocket teeth; in limit conditions, scuffing (seizure) may occur	Elongation of the chain and incorrect meshing of the sprocket with the chain	Limiting by calculation the pressure in the joints to values accepted by the lubricant film; improving the lubrication conditions
Breakage of the chain elements or of the sprocket teeth; cracking of the rollers	Removal from service	At the holes, in the case of heavily loaded chains operating at high speeds	Exceeding the tensile strength of the link plate material or the shear strength of the pin material; exceeding the bending strength of the pin, sprocket or bush materials	Interruption of the motion transmission	Limiting by calculation the tensile, shear and/or bending stresses to permissible values; limiting by calculation the operating speed to permissible values

TLA-T.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS



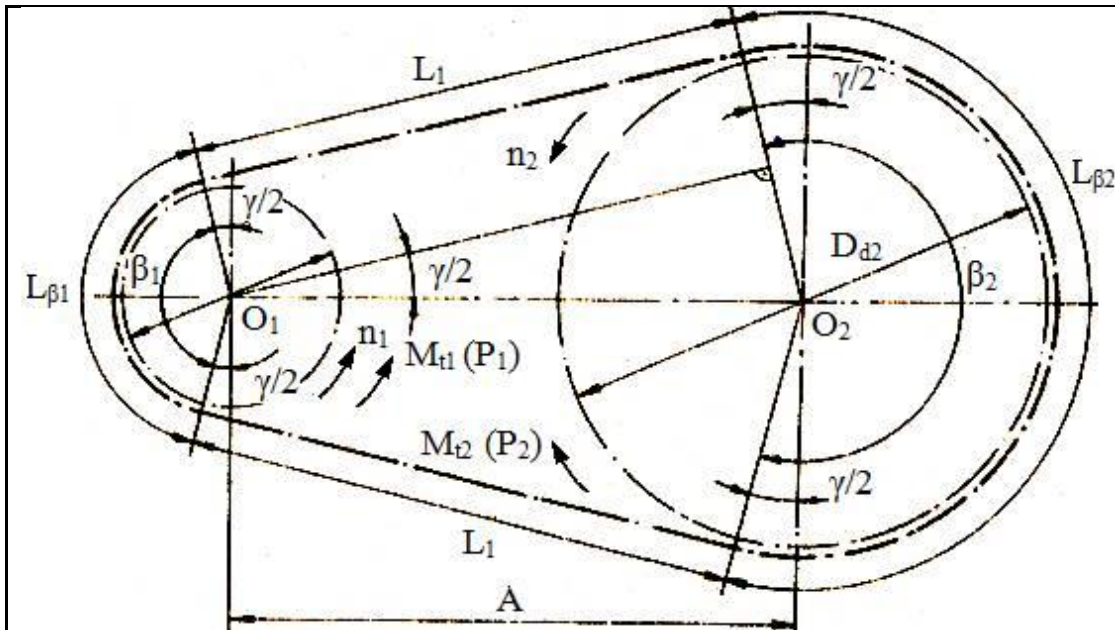


Fig. TLA-T.7.2 Geometric and kineto-static parameters of a chain drive

Main geometric parameters of the chain:

- pitch, p – the distance between the centres of two consecutive joints;
- number of joints (links), z ;
- chain length, $L = z p$;

Main geometric parameters of the sprockets (fig. TLA-T.7.1):

- chordal pitch, p (the same as the chain pitch);
- angular pitch, $\alpha_{1,2}$ – the central angle corresponding to the chordal pitch ($\alpha_{1,2} = 2\pi/z_{1,2}$);
- numbers of teeth of the sprockets (driving/driven), $z_{1,2}$;
- pitch circle diameters of the sprockets (driving/driven), $D_{d1,2}$ – the diameters of the circles on which the centres of the joints lie;
- outside diameters of the sprockets (driving/driven), $D_{e1,2}$;
- root diameters of the sprockets (driving/driven), $D_{i1,2}$;
- sprocket width, b .

Main geometric parameters of the chain drive (fig. TLA-T.7.2):

- centre distance, A ;
- wrap angles of the chain on the sprockets (driving/driven), $\beta_{1,2}$;
- angle between the chain spans, γ .

Main kineto-static parameters of the chain drive (fig. TLA-T.7.2):

- sprocket speeds (driving/driven), $n_{1,2}$;
- torques (powers) at the sprockets (driving/driven), $M_{t1,2} (P_{1,2})$;
- mean chain speed, v_m ;
- transmission ratio, $i = n_1/n_2 = D_{d2}/D_{d1} = z_2/z_1$

TLA-T.8 CALCULATION MODELS

TLA-T.8.1 CALCULATION MODELS OF CHAIN DRIVES WITHOUT TENSIONING SPROCKET

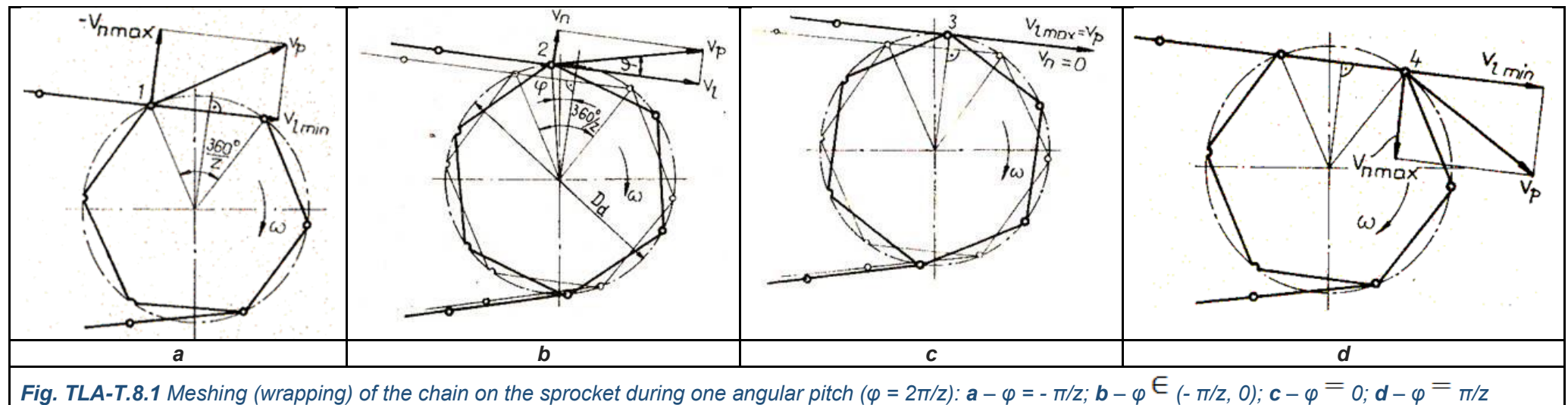
TLA-T.8.1.1 Calculation model of the geometric parameters

Tab. TLA-T.8.1 Calculation relations of the geometric and functional parameters

Calculation relations of the geometric parameters (fig. TLA-T.7.2)		
Pitch diameters of the sprockets	$D_{d1,2} = \frac{p}{\sin \frac{\pi}{z_{1,2}}}$, $D_{d1,2} \approx \frac{p}{\pi} z_{1,2}$	(TLA-T.8.1.1)
Angle between the spans of the drive	$\gamma = 2 \arcsin \frac{D_{d2} - D_{d1}}{2A}$; $\gamma \approx \frac{D_{d2} - D_{d1}}{2A} \approx \frac{p}{2\pi A} (z_2 - z_1)$	(TLA-T.8.1.2)

Wrap angles of the chain on the sprockets	$\beta_{1,2} = \pi \mp \gamma/2 = \pi \mp \arcsin \frac{D_{d2}-D_{d1}}{2A} \approx \pi \mp \frac{D_{d2}-D_{d1}}{A} = \pi \mp \frac{p}{\pi} \frac{z_2-z_1}{A}$ (the upper sign corresponds to the first index)	(TLA-T.8.1.3)
Chain length	$L = 2L_1 + L_{\beta 1} + L_{\beta 2} = 2A \cos \frac{\gamma}{2} + \beta_1 \frac{D_{d1}}{2} + \beta_2 \frac{D_{d2}}{2} =$ $2A \cos \frac{\gamma}{2} + \frac{p}{2} (z_2 + z_1) + \frac{1}{2A} \left(\frac{p}{\pi} (z_2 - z_1) \right)^2$	(TLA-T.8.1.4)
	Considering $\cos(\gamma/2) \approx 1 - (\gamma/2)^2/2$, it results $L \cong 2A + \frac{p}{2} (z_2 + z_1) + \frac{p}{A} \left(\frac{z_2 - z_1}{2\pi} \right)^2$	(TLA-T.8.1.5)
Number of links	$\frac{L}{p}$ $z = p$	(TLA-T.8.1.6)
Centre distance	$A = \frac{p}{4} \left[z - \frac{z_2 + z_1}{2} + \sqrt{\left(z - \frac{z_2 + z_1}{2} \right)^2 - 8 \left(\frac{z_2 - z_1}{2\pi} \right)^2} \right]$	(TLA-T.8.1.7)

TLA-T.8.1.2 Kinematic calculation model



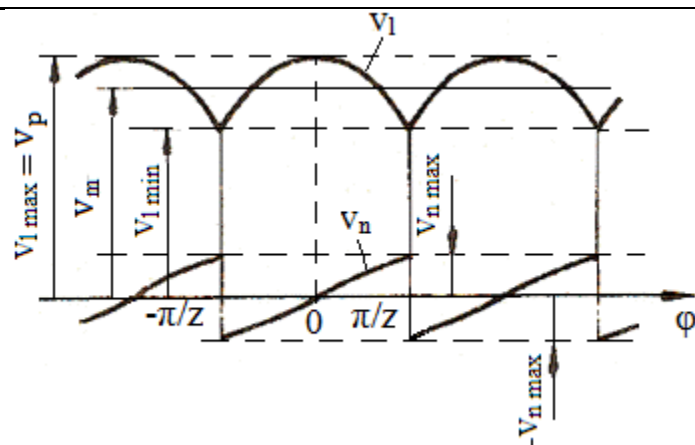


Fig. TLA-T.8.2 Variation of the velocity components of the joint

Assumptions:

- as a consequence of the polygonal wrapping of the chain, the meshing between the sprocket teeth and the chain links generates an oscillating motion about their own joints;
- the angular velocity of the sprocket, ω , is constant;
- the motion of the joint centre while the sprocket rotates through the angular pitch, $\alpha = 2\pi/z$, is considered as a displacement along two directions: longitudinal (along the chain span), with the velocity v_l , and normal to the chain span, with the velocity v_n ;
- the resultant velocity of the joint centre is equal to the peripheral velocity of the sprocket,

$$v_p = \sqrt{v_l^2 + v_n^2} = \frac{D_d}{2} \omega = ct.$$

Calculation relations of the velocities in the current point 2 (fig. TLA-T.8.1,b):

$$v_l = v_p \cos\varphi = \frac{D_d}{2} \omega \cos\varphi, \quad v_n = v_p \sin\varphi = \frac{D_d}{2} \omega \sin\varphi, \quad (\text{TLA-T.8.1.8})$$

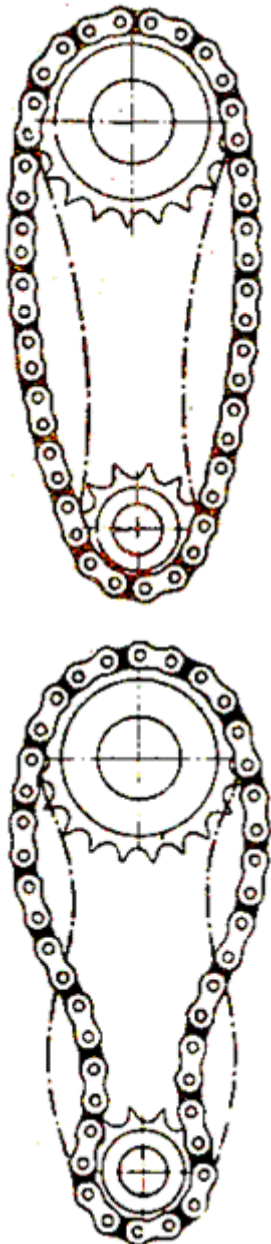


Fig. TLA-T.8.3 Modes of transverse oscillation of the spans of chain drives

Note.

1. At the entry into mesh, point 1 ($\varphi = -\pi/z$, fig. TLA-T.8.1,a), the longitudinal velocity,

$$v_l = \frac{D_d}{2} \omega \cos\left(-\frac{\pi}{z}\right) = \frac{D_d}{2} \omega \cos\left(\frac{\pi}{z}\right) = v_{l \min}$$

and the normal velocity,

$$v_n = \frac{D_d}{2} \omega \sin\left(-\frac{\pi}{z}\right) = -\frac{D_d}{2} \omega \sin\left(\frac{\pi}{z}\right) = -v_{n \max}$$

2. At point 3, after half a pitch ($\varphi = 0$, fig. TLA-T.8.1,c), the longitudinal velocity,

$$v_l = \frac{D_d}{2} \omega = v_{l \max} = v_p$$

and the normal velocity, $v_n = 0$.

3. At point 4, after one pitch ($\varphi = \pi/z$, fig. TLA-T.8.1,d), the longitudinal velocity,

$$v_l = \frac{D_d}{2} \omega \cos\left(\frac{\pi}{z}\right) = v_{l \min} \quad (\text{the same as at the entry into mesh, at point 1}),$$

and the normal velocity

$$v_n = \frac{D_d}{2} \omega \sin\left(\frac{\pi}{z}\right) = v_{n \max} \quad (\text{equal to the value at the entry into mesh, at point 1}).$$

4. The variation of the longitudinal velocity (along the chain span), v_l , is cyclic (fig. TLA-T.8.2), repeating at each rotation sequence equal to the angular pitch ($2\pi/z$); it produces accelerations and decelerations along the chain and, consequently, the driven sprocket will have a non-uniform rotational motion ($\omega_2 \neq \text{const.}$) with torsional oscillations, which introduces periodic dynamic torsional loads in the transmission.

5. The normal velocity (perpendicular to the chain span), v_n , varies with accelerations and decelerations and, therefore, with dynamic loads which generate transverse oscillations of the chain span (fig. TLA-T.8.3); when a new link comes into contact with the sprocket tooth, a shock occurs due to the jump of the velocity v_n from $-v_{n \max}$ to $v_{n \max}$.

6. Since the extreme values of the velocities v_l and v_n depend on the peripheral velocity ($v_p = \omega D_d/2$) and on the half-pitch of the chain (π/z_1), in order to reduce the dynamic loads (torsional and transverse) and thus to diminish vibrations (noise) and even wear, chains with pitches as small as possible and a limited chain speed are recommended (usually $v < 15$ m/s; max. 30 m/s for high-quality chains); to avoid transverse oscillations (vibrations), a chain tensioning system with a sprocket (fig. TLA-T.2.1,a) and/or guidance with a shoe (fig. TLA-T.2.1,b) is adopted.

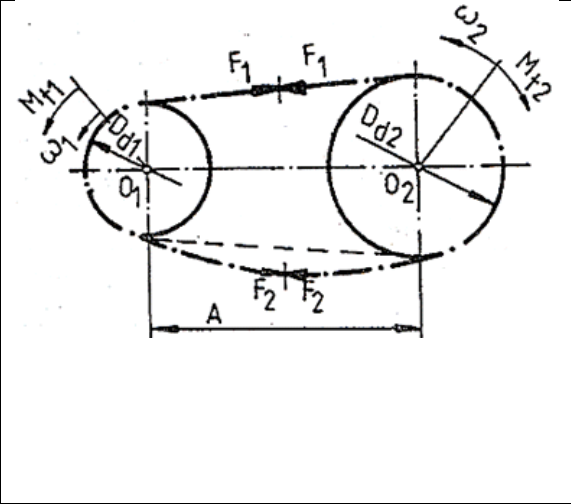
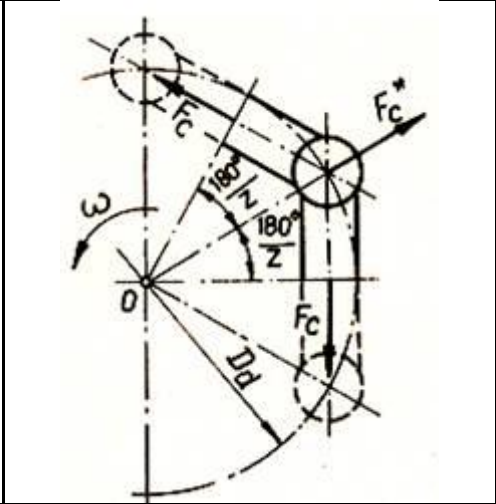
7. The mean speed of the chain, assuming a uniform displacement along the perimeter of the pitch polygon, is determined with the relation

$$V_m = \omega \frac{D_d}{2 \cdot 1000} = \frac{\pi n}{30} \frac{D_d}{2 \cdot 1000} = \frac{\pi n}{30} \frac{p}{2 \cdot 1000 \sin \frac{\pi}{z}} \approx \frac{z p n}{60 \cdot 1000} \quad [\text{m/s}], \quad (\text{TLA-T.8.1.9})$$

where p [mm] is the pitch, ω_1/n_1 – the angular velocity/speed [rad/s / rpm] of sprocket 1 and z – the number of teeth of sprocket 1;

D_d [mm] – the pitch diameter of the sprocket

TLA-T.8.1.3 Kineto-static calculation model

		Assumptions: - the dynamic effects generated by the polygonal action (fig. TLA-T.8.1) are neglected, - friction is neglected. Calculation relations The maximum total force in the driving (tight) span, F_1, from the equilibrium condition of the driving sprocket, $F_1 \frac{D_{d1}}{2} - F_2 \frac{D_{d1}}{2} = M_{t1}, \quad (\text{TLA-T.8.1.10})$ $F_u = \frac{2 M_{t1}}{D_{d1}}$ is $F_1 = F_2 + F_u$, where F_u is the useful force (to be transmitted). The tensioning force in the slack span, $F_2 = F_q + F_c, \quad (\text{TLA-T.8.1.11})$ where F_q is the tensioning force due to the own weight of the links of the slack span of the chain and F_c is the force due to the centrifugal forces of the chain portions wrapped on the sprockets. The tensioning force due to the own weight of the links of the slack span in a horizontal position, $F_q \approx q A, \quad (\text{TLA-T.8.1.12})$ where q is the weight per unit length of the chain and A – the centre distance. $F_c = \frac{F_c^*}{2 \sin(\frac{\pi}{Z})} = \frac{m \frac{v_m^2}{D_{d1}/2}}{2 \sin(\frac{\pi}{Z})} = \frac{m \frac{v_m^2}{p/\sin(\frac{\pi}{Z})}}{\sin(\frac{\pi}{Z})} = m \frac{v_m^2}{p} = \frac{q v_m^2}{g}, \quad (\text{TLA-T.8.1.13})$ where v_m is the mean speed of the chain (TLA-T.8.1.9), p – the chain pitch and $m = p q/g$, the mass of one link (g – gravitational acceleration)
a	b	
Fig. TLA-T.8.4 Forces in chain drives: a – general scheme; b – detail for the centrifugal force		
Note. The force loading the shafts of the chain drive is $F_a = k_g F_u$, where $k_g = 1.05$ for horizontal drives, $k_g = 1.2$ for drives with the axes in a vertical plane, and intermediate values for other inclinations of the line of centres.		

TLA-T.8.1.4 Strength calculation model

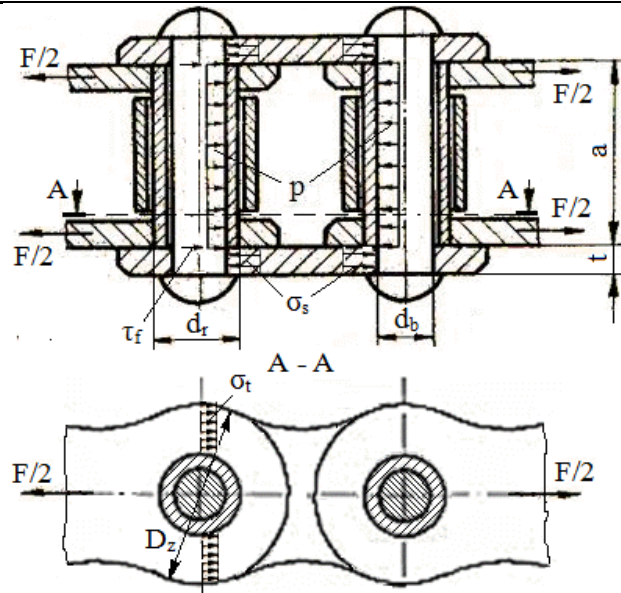


Fig. TLA-T.8.5 Strength calculation scheme

Assumptions:

- when manufacturing and assembly comply with the technical documentation, the main cause of failure is considered to be the wear of the chain joints (it leads to chain elongation and improper operation);
- the wear resistance condition is expressed by the condition that the lubricant film between pin and bush or between bush and roller is not crushed (squeezed out);
- the total force in the tight span is taken by the first tooth; dynamic loads are neglected.

Crushing calculation:

Mean pressure in the pin-bush contact zone (fig. TLA-T.8.5),

$$p = k_e \frac{F}{a d_b} \leq p_a \quad . \text{ (TLA-T.8.1.13)}$$

Bearing (crushing) stress in the pin – outer link plate zone (fig. TLA-T.8.5),

$$\sigma_s = \frac{\frac{F}{2}}{d_b t} = \frac{F}{2 d_b t} \leq \sigma_{as} \quad . \text{ (TLA-T.8.1.14)}$$

Shear calculation of the pin,

$$\tau_f = \frac{\frac{F}{2}}{\frac{\pi d_b^2}{4}} = \frac{2 F}{\pi d_b^2} \leq \tau_{af} \quad . \text{ (TLA-T.8.1.15)}$$

Tensile calculation of the inner link plate in the bush zone

$$\sigma_t = \frac{\frac{F}{2}}{t (D_z - d_r)} = \frac{F}{2 t (D_z - d_r)} \leq \sigma_{at} \quad . \text{ (TLA-T.8.1.16)}$$

Meanings of the parameters in relations (TLA-T.8.1.13...16): $F = F_1$, the total force in the tight span; d_b – pin diameter; a – bush length; t – link plate thickness; D_z – diameter of the inner link plate in the zone where it is fixed on the bush; d_r – outer diameter of the bush (inner diameter of the roller); p_a – permissible crushing pressure of the lubricant (10...15 MPa), established experimentally for average operating conditions (chosen depending on the pitch and on the speed of the small sprocket); $\sigma_{as} = \min (\sigma_{asb}, \sigma_{ase})$, with σ_{asb} the permissible bearing stress of the pin material and σ_{ase} the permissible bearing stress of the link plate material; τ_{af} – permissible shear stress of the pin material; σ_{at} – permissible tensile stress of the link plate material; k_e – service factor, whose purpose is to adapt the data of the drive being calculated to the data of the reference drive for which the permissible pressure was determined experimentally.

Note. The service factor is recommended by chain manufacturers as a product, $k_e = k_d k_a k_i k_r k_f$, whose components depend on the nature of the transmitted load (constant or with shocks), the centre distance, the inclination of the line of centres, the sprocket adjustment system (by moving one sprocket, pressure roller), the lubrication system (drip, oil bath, etc.) and, respectively, the operating time (number of work shifts).

TFR. FRICTION DRIVES

TFR-T. CONSTRUCTION AND MODELING OF FRICTION DRIVES

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TFR-T.1 FRICTION DRIVES WITH CONSTANT TRANSMISSION RATIO

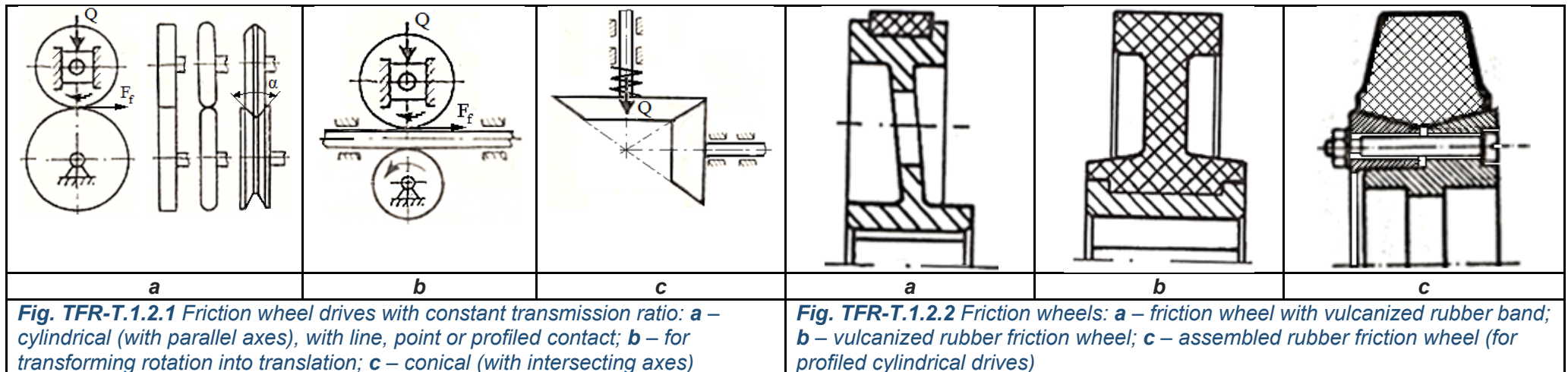
TFR-T.1.1 DEFINITION

Friction drives transmit the rotational motion and the torque from a driving element to one or more driven elements by means of the friction force, while the wheel-type active elements, in direct contact, are pressed against each other with a normal force large enough to avoid slipping. Considering the type of the active elements, these drives are also known as *friction wheel drives*.

Friction drives with constant transmission ratio are friction wheel drives in which the rotational motion and the torque are transmitted from the driving element to the driven one, which runs at a quasi-constant speed; they have no possibility of changing the transmission ratio.

TFR-T.1.2 CONSTRUCTIVE STRUCTURES

Friction wheel drives with constant transmission ratio can involve rotational motions only (fig. TFR-T.1.2.1,a,c) or also translational motions (fig. TFR-T.1.2.1,b). To reduce the pressing force Q (reducing the wear and the loads of the adjacent elements), in the case of cylindrical drives, profiled wheels can be adopted (fig. TFR-T.1.2.1,a, right-hand variant); they have conical contact surfaces and ensure a wedge effect, equivalent to a substantial increase of the coefficient of friction μ (contact between cylindrical surfaces) to the value $\mu/\sin(\alpha/2)$.



TFR-T.1.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Main areas of use:

- lightly loaded drives, especially with a kinematic role;

- drives that require shock-free engagement/disengagement of the elements;
- drives loaded with relatively small loads, but which must operate at very high speeds or with very low noise and vibration levels.

Tab. TFR-T.1.3.1 Advantages and disadvantages of friction drives with constant transmission ratio

Advantages*	Disadvantages*
- operation with low vibration and noise levels; - protection of the drive against overloads occurring in operation; - constructively simple solutions; - the drive can be used as a shock-free engaging/disengaging system during motion; - relatively low costs (for some solutions the cost is about 50% of that of a gear drive).	- the transmission ratio is not kept constant during operation, because of the (geometric or elastic) slip in the contact zone and of the manufacturing and assembly errors; - lower efficiency, especially compared to gear drives; - increased and non-uniform wear of the active elements due to slip in the contact zone; - relatively low durability; - high pressing forces are required, which load the shafts and bearings and, therefore, lead to larger overall dimensions
* Compared to other mechanical drives (gear drives, belt drives, etc.)	

TFR-T.1.4 CLASSIFICATION

Tab. TFR-T.1.4.1 Classification of friction drives with constant transmission ratio

Criterion	Type	Scheme
Position of the friction wheel axes	Parallel	Fig. TFR-T.1.7.1.2,a,b
	Intersecting	Fig. TFR-T.1.7.1.2,c
Type of contact	Line contact	Fig. TFR-T.1.7.1.2
	Point contact	Fig. TFR-T.1.2.1,a (middle variant)
Type of pressing subsystem	With constant pressing force	Fig. TFR-T.1.7.2.1
	With automatic (variable) pressing force	Fig. TFR-T.1.7.2.2, TFR-T.1.7.2.3

TFR-T.1.5 MATERIALS AND TECHNOLOGIES

The *main requirements* that the materials used for the active elements of friction drives must meet are: contact strength; wear resistance; a coefficient of friction as high as possible and constant in time.

Pairs of metallic materials characterized by high contact strength and wear resistance can be:

- hardened steel / hardened steel – for heavily loaded drives, which require high durability and operate with or without lubrication – characterized by minimum overall size and high efficiency; they require high manufacturing and assembly accuracy, together with the reduction of the geometric slip, which could lead to scuffing;
- cast iron / hardened steel – for drives operating with or without lubrication, with the advantage of increased scuffing resistance;
- cast iron / cast iron – for drives operating with lubrication.

Pairs of non-metallic/metallic materials characterized by high coefficients of friction and increased elasticity – textolite, rubber, leather, etc./steel or cast iron – for lightly loaded drives, operating without lubrication, characterized by large overall dimensions and lower efficiency; the non-metallic material is usually used as linings, mounted (glued, vulcanized) on the driving element, to ensure uniform wear.

TFR-T.1.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. TFR-T.1.6.1 Forms and causes of improper behavior or removal from service

Forms	Consequences	Occurrence	Causes	Manifestation	Avoidance
Damage of the surfaces of the active elements by pitting	Improper behavior (with irregularities, vibrations and noise)	Lubricated drives	Contact fatigue (pitting)	Microcracks and pits on the active surfaces	Limiting the maximum contact stress to values accepted by the pair of materials
Damage of the surfaces of the active elements by scuffing	Improper behavior (with irregularities, vibrations and noise)	Drives with improper lubrication at high loads and speeds	Scuffing	Increase of friction and of local temperatures; microwelds and scratches on the active surfaces	Limiting the operating speed to permissible values
Damage of the surfaces of the active elements by abrasive wear	Improper behavior (with irregularities, vibrations and noise)	Heavily loaded high-speed drives without lubrication and/or unsealed	Abrasive wear	Scratches on the active surfaces	Proper lubrication and sealing

TFR-T.1.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

TFR-T.1.7.1 Functional parameters

The process of transmitting energy by friction is always accompanied by *elastic slip* and *geometric slip*.

Elastic slip is caused by the temporary change (only during contact) of the geometry of the contact zone, which implies the transformation (elastic deformation) of the theoretical (line or point) contact into a surface, as a result of the normal pressing force Q and of the useful force F_t transmitting the load.

The *elastic slip factor* can be determined with the relation,

$$\xi_e = \frac{v_{al}}{v_1} = \frac{v_1 - v_2}{v_1} = \frac{\omega_1 r_1 - \omega_2 r_2}{\omega_1 r_1} = 1 - \frac{r_2}{i r_1}, \quad (\text{TFR-T.1.7.1.1})$$

from which the transmission ratio is determined,

$$i = \frac{r_2}{r_1} \frac{1}{1 - \xi_e}, \quad (\text{TFR-T.1.7.1.2})$$

where $i = \omega_1/\omega_2$ is the transmission ratio; $r_{1,2}$ – the radii of bodies 1, 2 at a point of the contact line; $v_{1,2}$ – the peripheral speeds of a point on the contact line.

Geometric slip occurs in friction wheel drives with line contact in which the contact does not take place along the instantaneous axis of rotation. Thus, in the case of bevel friction wheels whose apexes do not coincide (fig. TFR-T.1.7.1.1), the speeds of the various contact points vary linearly (section X - X, fig. TFR-T.1.7.1.1), and there is a single point, the pole of relative motion P, at which the speeds are equal (the circles of the two wheels tangent at P roll without slipping relative to each other, so at this point the relative sliding velocity is zero). At all the other contact points the tangential speeds are different and slip occurs, with the maximum sliding velocity at point B, $v_{al \max} = v_{B1} - v_{B2}$, where $B_1 \in 1$ and $B_2 \in 2$ (points $B_{1,2}$ coincide with B).

The *geometric slip factor* is defined as the absolute value of the ratio between the maximum sliding velocity and the peripheral velocity of the pole of motion,

The value of the elastic slip factor depends on the characteristics of the wheel materials:

$\xi_e \leq 0.002$, when both wheels are made of steel; $\xi_e \leq 0.02$, when one wheel is made of steel and the other is lined with a plastic material (textolite).

The factor ξ_e depends on the magnitude of the transmitted load.

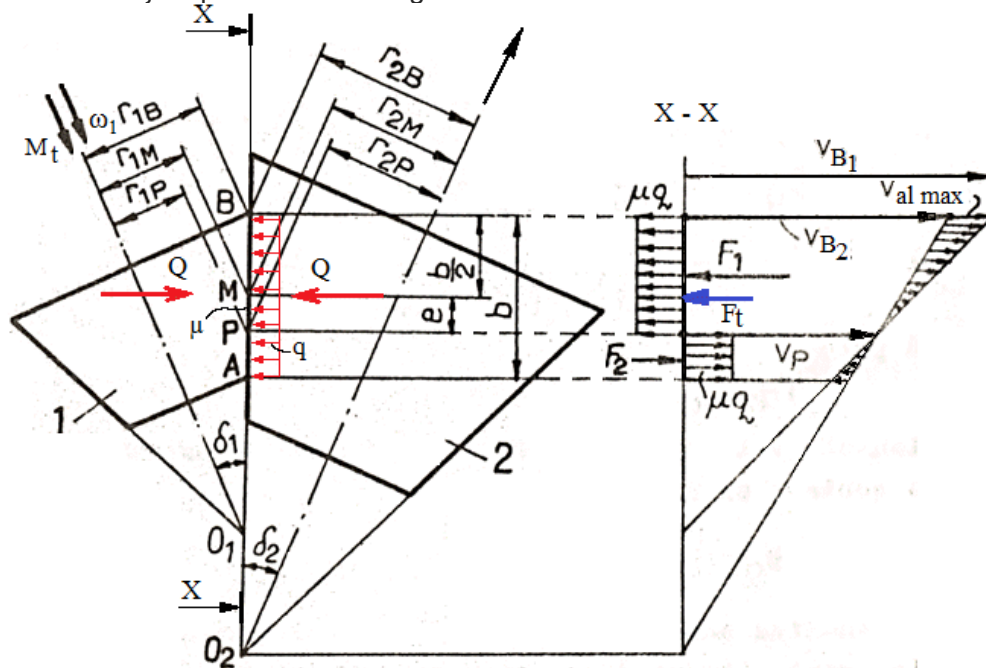


Fig. TFR-T.1.7.1.1 Calculation scheme of the geometric slip

$$\xi_g = \left| \frac{v_{al\ max}}{V_p} \right| = \left| \frac{\omega_1 r_{1B} - \omega_2 r_{2B}}{\omega_1 r_{1P}} \right| = \left| \frac{i r_{1B} - r_{2B}}{i r_{1P}} \right| = \left| \frac{i r_{1B} - r_{2B}}{i (r_{1B} - (e + b/2) \sin \delta_1)} \right|$$

(TFR-T.1.7.1.3)

where $\omega_{1,2}$ are the angular velocities of the two cones, $i = \omega_1/\omega_2$ – the transmission ratio; r_{1B} , r_{2B} – the radii of bodies 1, 2 at point B; b – the length of the contact line; e – the distance between the pole of relative motion P and the centre of the contact line M; δ_1 – the half-angle of cone 1.

The condition for transmitting the torque from wheel 1 to wheel 2 by friction can be expressed by the relations:

$$M_{tf} > M_t, M_{tf} = c M_t, \text{ or } \mu Q > F_t, \mu Q = c F_t, \text{ (TFR-T.1.7.1.4)}$$

where M_{tf} is the friction torque resulting from the forced contact of the two wheels under the pressing force Q , normal to the contact surfaces; M_t – the torque at wheel 1; F_t – the useful tangential force; c – the safety coefficient against slipping; μ – the coefficient of friction of the pair of materials in contact. As a result of the pressing force Q , along the contact line of length b , a distributed force appears,

$$q = \frac{Q}{b} = \text{const.} \quad \text{(TFR-T.1.7.1.5)}$$

The sliding velocities, whose direction changes at pole P, imply distributed friction forces μq , also of opposite directions (fig. TFR-T.1.7.1.1), with the resultants:

$$F_1 = \mu q (0.5b + e), F_2 = \mu q (0.5b - e). \quad \text{(TFR-T.1.7.1.6)}$$

Thus, the useful tangential force,

$$F_t = F_1 - F_2 = 2\mu q e = 2\mu Q e/b. \quad \text{(TFR-T.1.7.1.7)}$$

Substituting this relation into the last relation of group (TFR-T.1.7.1.4), one obtains,

$$e = \frac{b}{2} \frac{F_t}{\mu Q} = \frac{b}{2c}. \quad \text{(TFR-T.1.7.1.8)}$$

These relations show that when the maximum torque is transmitted (the tangential force F_t is maximum), the safety factor against slipping is minimum, $c = 1$, and the distance between pole P and centre M is maximum, $e = b/2$; in idle operation, $F_t = 0$, $c \rightarrow \infty$, this distance is zero, $e \rightarrow 0$; in all other cases, $c > 1$, $e < b/2$.

In practice, constant-ratio drives are frequently used to transmit the torque and the rotational motion between parallel axes (fig. TFR-T.1.7.1.2,a,b) or intersecting axes (fig. TFR-T.1.7.1.2,c).

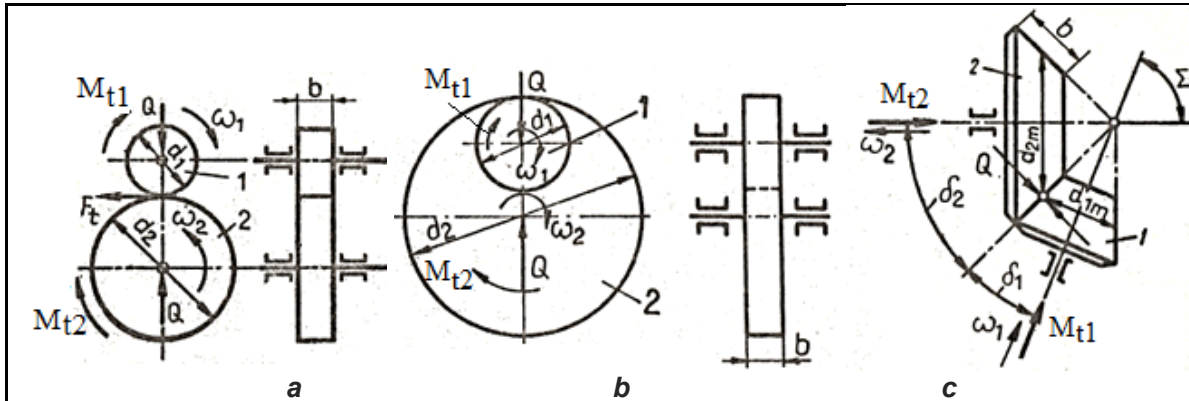


Fig. TFR-T.1.7.1.2 Schemes of friction drives with constant transmission ratio: **a, b** – with parallel axes (cylindrical); **c** – with intersecting axes (bevel)

Transmission ratio

$$i = \frac{\omega_1}{\omega_2} = \frac{1}{1 - \xi_e} \frac{d_2}{d_1} \quad (\text{TFR-T.1.7.1.9})$$

for drives with parallel axes,

$$i = \frac{\omega_1}{\omega_2} = \frac{1}{1 - \xi_e} \frac{\sin \delta_2}{\sin \delta_1}, \quad (\text{TFR-T.1.7.1.10})$$

for drives with intersecting axes, where $\omega_{1,2}$ are the angular velocities of the wheels; ξ_e – the elastic slip factor; $d_{1,2}$ – the diameters of the cylindrical wheels; $\delta_{1,2}$ – the half-angles of the bevel wheels.

TFR-T.1.7.2 Constructive parameters. Pressing subsystems

Transmitting mechanical energy by friction requires a permanent and forced contact of the friction wheels, which is achieved by pressing subsystems. Depending on how the pressing force varies in time, there are constant-force pressing subsystems, in which the pressing force is constant, independent of the transmitted torque, and automatic (variable-force) pressing subsystems, in which the pressing force changes continuously (automatically) depending on the transmitted torque.

Constant-force pressing subsystems

The schemes of the constant-force pressing subsystems (fig. TFR-T.1.7.2.1) show relative displacements of the axis of one wheel (often the driving one), which allow direct loading of the contact zone with the force,

$$Q = G \frac{a+b}{a}, \text{ (TFR-T.1.7.2.1)}$$

$$Q = k \delta_0, \text{ (TFR-T.1.7.2.2)}$$

$$Q = p A_p, \text{ (TFR-T.1.7.2.3)}$$

generated by the weight G through the lever mechanism with arms of lengths a and b (fig. TFR-T.1.7.2.1,a), by the elastic force of a preloaded spring of stiffness k (fig. TFR-T.1.7.2.1,b) or, respectively, by the hydrostatic/pneumostatic force generated by the hydraulic/pneumatic motor with cylinder area A_p and fluid pressure p (fig. TFR-T.1.7.2.1,c).

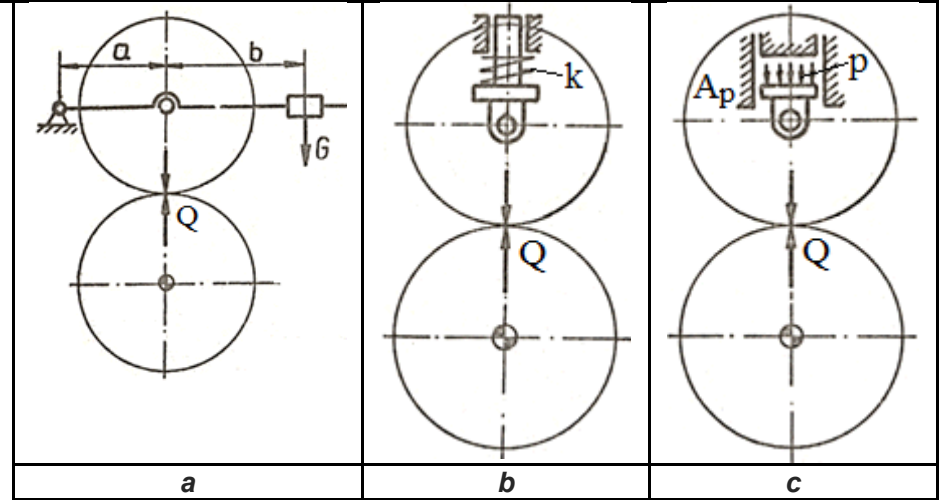


Fig. TFR-T.1.7.2.1 Constant-force pressing systems: **a** – gravitational; **b** – elastic; **c** – hydrostatic or pneumostatic

Characteristics of the constant-force pressing systems:

- a.** Since the pressing force always has the maximum value (corresponding to the maximum transmitted torque), independent of the transmitted torque, it permanently loads the elements of the drive (shafts, bearings, housing, etc.) at the maximum level and leads, on the one hand, to larger overall dimensions of the drive and, on the other hand, to reduced durability of the friction wheels and rolling bearings, also having a negative influence on the efficiency of the drive (especially at maximum load).
- b.** They also act as load limiters of the drive.
- c.** They are constructively simple and are usually recommended for low-power friction drives.

Automatic (variable-force) pressing subsystems

Automatic pressing systems eliminate the disadvantages of constant-force pressing systems, but they do not act as load limiters of the drive.

$$Q = \frac{F_a}{\sin \delta} = \frac{F_{t1}}{\sin \delta \operatorname{tg}(\beta + \varphi)} = \frac{2 M_t}{d_a \sin \delta \operatorname{tg}(\beta + \varphi)} = \frac{2 F_t r_M}{d_a \sin \delta \operatorname{tg}(\beta + \varphi)} \quad (\text{TFR-T.1.7.2.4})$$

where M_t [Nmm] is the transmitted torque, F_{t1} [N] – the tangential force in the pressing subsystem, F_t [N] – the useful tangential force in the contact zone (point M), d_a [mm] – the diameter of the contact points of the face (dog) teeth, the diameter of the ball centres, the pitch diameter of the helical gear or the mean diameter of the thread; r_M [mm] – the radius of the friction wheel at the contact point M; β [°] – the inclination angle of the dogs, the inclination angle of the grooves in which the balls are mounted, the complement of the helix angle of the teeth or the lead angle of the thread; $\varphi = \arctg \mu$ [°] – the friction angle corresponding to sliding friction; μ – the sliding friction coefficient.

Note.

- a.** in steady-state operation ($M_t = \text{const.}$), $\varphi = 0$ is considered in the above relation;
- b.** relation (TFR-T.1.7.2.4) shows that the ratio Q/F_t changes with the radius of the friction wheel at the contact point, r_M ;
- c.** to avoid slipping of the drive during start-up, additional springs are introduced in the construction of the pressing subsystems, preloaded with a force that ensures a minimum pressing force at start-up;
- d.** fig. TFR-T.1.7.2.3 shows a constructive solution of a pressing subsystem with a ball coupling, frequently integrated in friction drives.

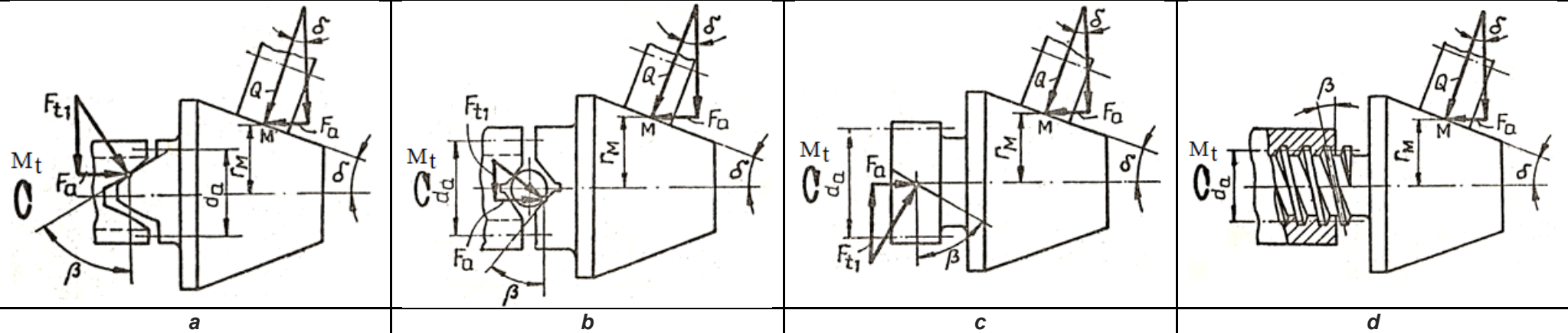


Fig. TFR-T.1.7.2.2 Automatic pressing subsystems for friction drives with intersecting or crossed axes: **a** – with face-tooth (dog) coupling; **b** – with ball coupling; **c** – with helical cylindrical gear; **d** – with helical (screw) pair

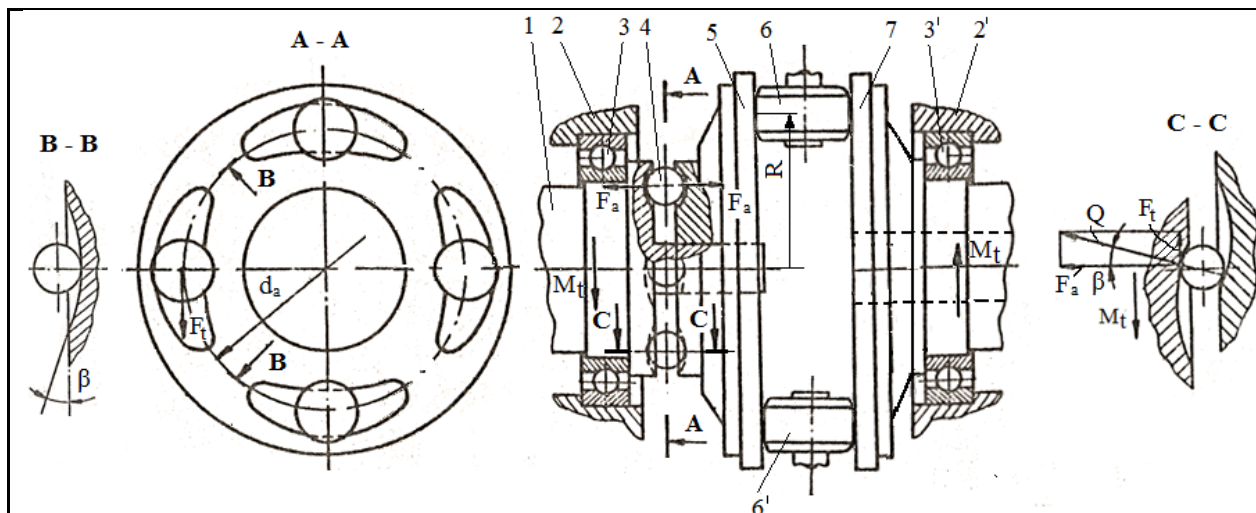


Fig. TFR-T.1.7.2.3 Automatic pressing subsystem with ball coupling integrated in a friction drive with discs and rollers

The torque M_t from the input shaft 1 is transmitted by shape to the balls 3, by shape to the driving friction disc 5, by friction to the rollers 6, 6' and by friction to the driven friction disc 7.

The axial force F_a (generated in the ball coupling) is transmitted, on the one hand, from the input shaft 1 through the radial ball bearing 3 to the half-housing 2 and, on the other hand, from the driving friction disc 5 to the rollers 6, 6', to the driven friction disc 7 and through the radial ball bearing 3' to the half-housing 2'.

The condition for transmitting the torque by friction is, $\mu Q R = c M_t$, (TFR-T.1.7.2.5) where M_t [Nmm] is the transmitted torque, Q [N] – the pressing force, R [mm] – the radius of the contacts, μ – the sliding friction coefficient, c – the safety coefficient against slipping.

From relation (TFR-T.1.7.2.4), considering $\delta = 90^\circ$ and taking into account relation (TFR-T.1.7.2.5), the pressure angle results,

$$\beta = \arctan \left(\mu \frac{2 R}{c d_a} \right) - \varphi \quad . \text{ (TFR-T.1.7.2.6)}$$

TFR-T.1.8 CALCULATION MODELS

TFR-T.1.8.1 Contact stress calculation model

For friction drives operating with lubrication, the main form of damage is contact fatigue (pitting) and, to avoid it, the contact stress calculation is necessary.

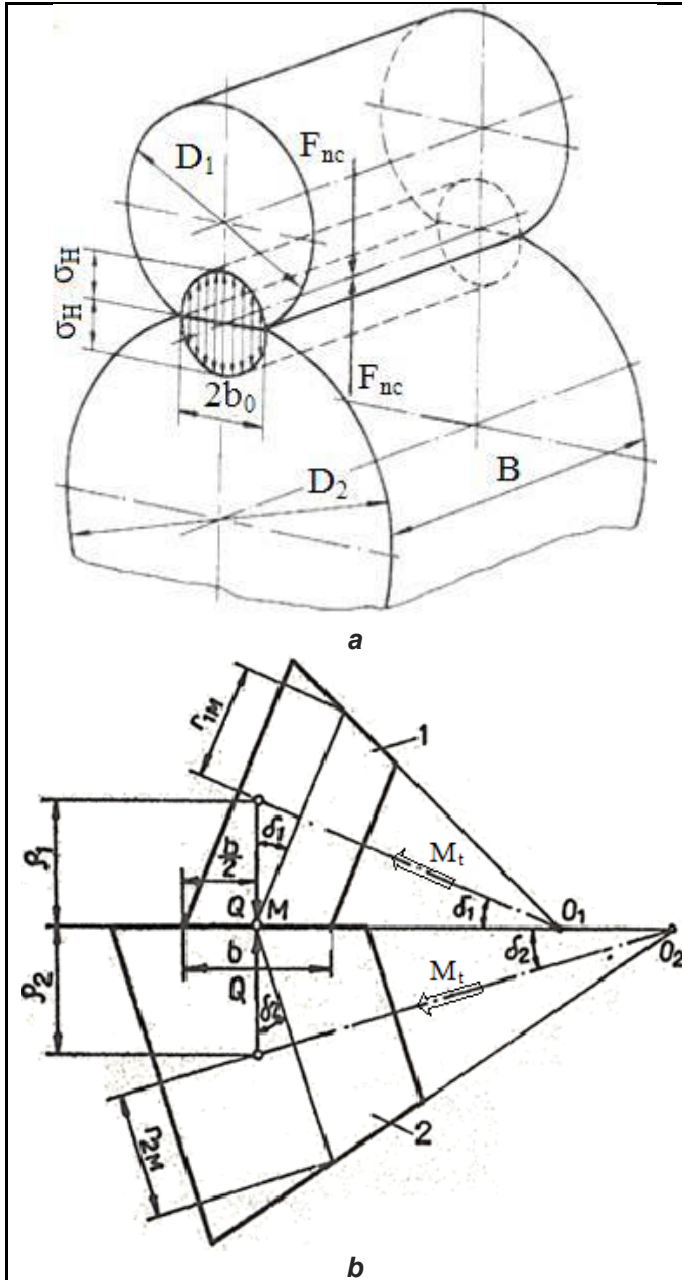


Fig. TFR-T.1.8.1 Contact calculation scheme: **a** – Hertz model; **b** – contact calculation model of the contact between two conical rollers of a friction drive

Reference calculation model for line contact stress [Hertz, 1881] (fig. TFR-T.1.8.1,a)

In the contact zone of two cylinders with diameters $D_{1,2}$ and lengths B , loaded with the force F_{nc} , maximum normal contact stresses occur, which, according to Hertz, are determined with the relation,

$$\sigma_H = Z_E \sqrt{\frac{F_{nc}}{l_k} \frac{1}{\rho}} \leq \sigma_{HP}, \quad (\text{TFR-T.1.8.1.1})$$

where $l_k = B$ is the length of the contact line;

$$\frac{1}{\rho} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{1}{D_1/2} \pm \frac{1}{D_2/2},$$

the reduced curvature, where $\rho_{1,2}$ are the radii of curvature of the two cylinders (the sign (+) corresponds to external contact (fig. AEC-T.1,a) and the sign (-) to internal contact);

$$Z_E = \sqrt{\frac{1}{\pi \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)}}$$

the elasticity factor of the materials, where $E_{1,2}$ and $v_{1,2}$ are the moduli of elasticity and, respectively, the transverse contraction (Poisson) coefficients of the materials of the two cylinders, and

$$\sigma_{HP} = \frac{\sigma_{Hlim}}{S_{Hmin}},$$

the permissible contact stress, where σ_{Hlim} is the limit contact stress and S_{Hmin} – the minimum safety coefficient against contact fatigue of the material.

Assumptions of the Hertz model: the materials of the bodies are homogeneous, isotropic and elastic according to Hooke's law; the normal force is applied statically; the contact stresses are uniformly distributed along the contact length; the surfaces of the cylinders are perfectly smooth; friction forces are not taken into account.

Contact stress calculation of friction drives with constant transmission ratio

Fig. TFR-T.1.8.1,b shows the general case of line contact between 2 conical rollers with non-intersecting axes. To calculate the maximum contact stress, the two surfaces are replaced by cylindrical surfaces with radii $\rho_{1,2}$ at the median point M (fig. TFR-T.1.8.1,b), in contact along the common generatrix under the load $F_{nc} = Q$.

Particularization of the Hertz model parameters for the contact along the common generatrix of the cones (TFR-T.1.8.1.1):

$$l_k = b,$$

$$F_{nc} = Q = \frac{c F_t}{\mu z} = \frac{c M_t}{\mu z r_{1M}},$$

$$\frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{\cos \delta_1}{r_{1M}} + \frac{\cos \delta_2}{r_{2M}} \cong \frac{i \cos \delta_1 + \cos \delta_2}{i},$$

where $i \approx r_{2M}/r_{1M}$ is the transmission ratio when the elastic and geometric slips are neglected, with r_{1M} , r_{2M} – the mean radii of the wheels (at the middle of the contact line); $\delta_{1,2}$ – the half-angles of the bevel friction wheels; μ – the sliding friction coefficient; c – the safety coefficient against slipping; z – the number of wheels driven by the driving wheel.

Thus, the calculation relation (TFR-T.1.8.1.1) becomes,

$$\sigma_H = \frac{Z_E}{r_{1M}} \sqrt{\frac{c M_t}{\mu z b} \frac{i \cos \delta_1 + \cos \delta_2}{i}} \leq \sigma_{HP}, \quad (\text{TFR-T.1.8.1.2})$$

and can be used to verify friction wheel drives with constant transmission ratio for contact stress. The second part of this relation can be used for the sizing calculation.

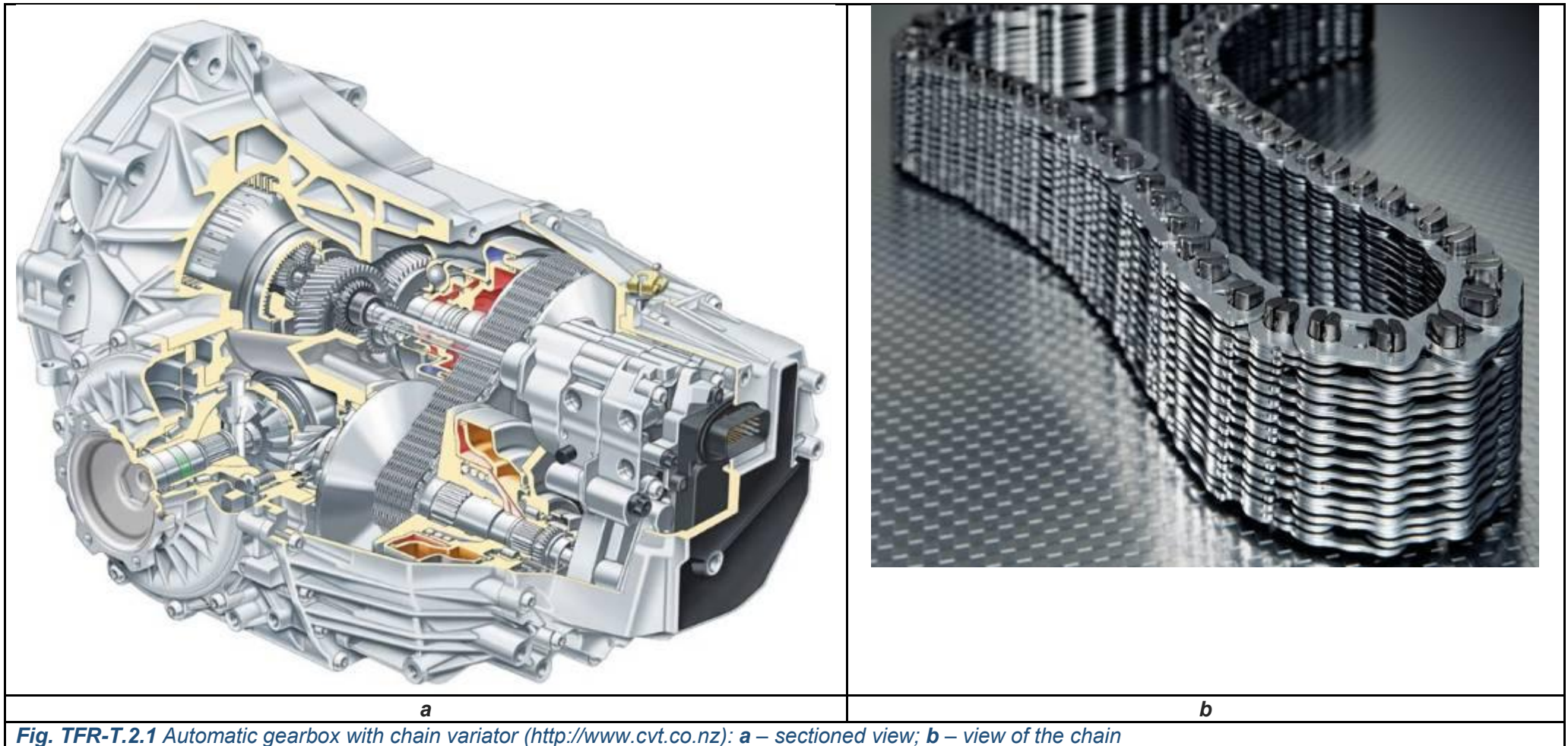
TFR-T.2 FRICTION DRIVES WITH VARIABLE TRANSMISSION RATIO (VARIATORS)

TFR-T.2.1 DEFINITION

Friction drives transmit the rotational motion and the torque from a driving element to one or more driven elements by means of the friction force, while the wheel-type active elements, in direct contact, are pressed against each other with a normal force large enough to avoid slipping.

Friction drives with variable transmission ratio are mechanical drives in which the rotational motion and the torque are transmitted from the driving element to the driven one, which runs at a speed that can be varied continuously between certain limits, since the transmission ratio can be changed continuously during motion.

TFR-T.2.2 CONSTRUCTIVE STRUCTURES



TFR-T.2.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Main areas of use: automatic gearboxes (CVT – Continuously Variable Transmission) of vehicles, agricultural machines, tractors, technological equipment.

Tab. TFR-T.2.3.1 Advantages and disadvantages of speed variators

Advantages*	Disadvantages*
- a constant torque transmission characteristic can be obtained; - operation with low vibration and noise levels;	- for a set transmission ratio, the ratio is not kept constant during operation because of the (geometric or elastic) slip in the contact zone;

- protection of the drive against overloads occurring in operation

- lower efficiency;
- non-uniform wear of the elements in contact;
- low durability;
- high pressing forces are required, which load the shafts and bearings and, therefore, lead to larger overall dimensions

* Compared to drives with constant transmission ratio (gear drives, belt drives, etc.)

TFR-T.2.4 CLASSIFICATION

Tab. TFR-T.2.4.1 Classification of friction variators

Criterion	Type	Examples
Number of contacts	Mono	Fig. TFR-T.2.4.1; TFR-T.2.8.1.1.1,a
	Duo	Fig. TFR-T.2.4.2; TFR-T.2.8.1.1.1,b
Position/shape of the active elements	Face (frontal)	Fig. TFR-T.2.4.1,a; TFR-T.2.8.1.1.1
	Conical	Fig. TFR-T.2.4.1,b
	Spherical	Fig. TFR-T.2.4.2,d; TFR-T.2.8.1.2.1
	Disc	Fig. TFR-T.2.4.1,c
Subsystem generating the pressing force	With constant pressing force	Fig. TFR-T.2.8.1.1.1
	With automatic (load-dependent) pressing force	Fig. TFR-T.2.8.1.3.1

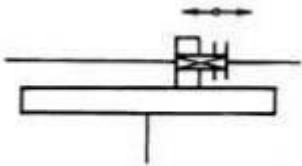
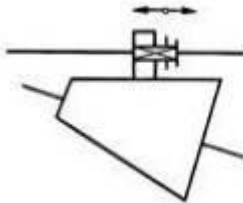
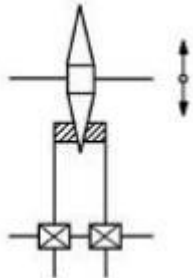
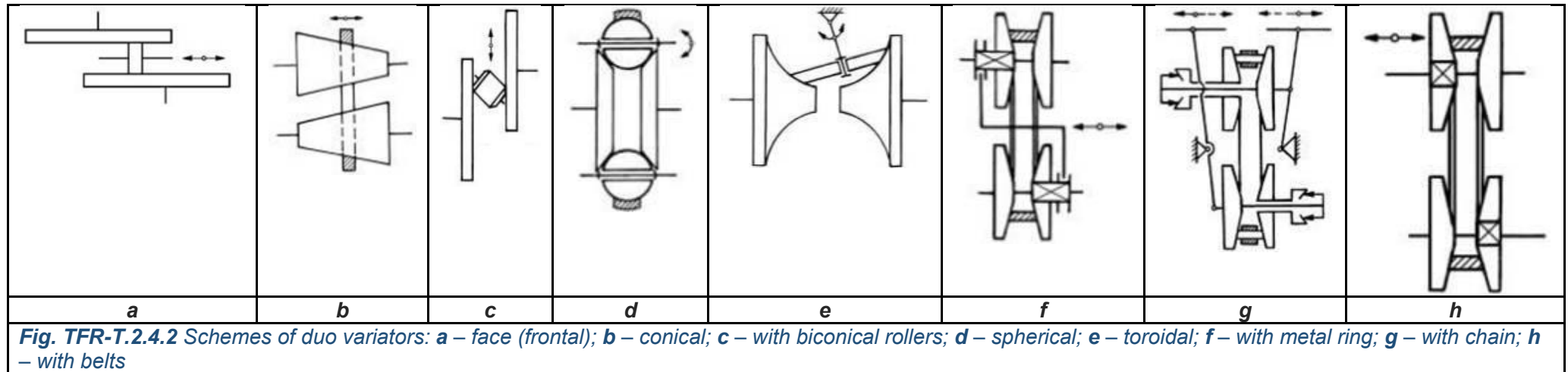
		
a	b	c

Fig. TFR-T.2.4.1 Schemes of mono variators: **a** – face (frontal); **b** – conical; **c** – with disc



TFR-T.2.5 MATERIALS AND TECHNOLOGIES

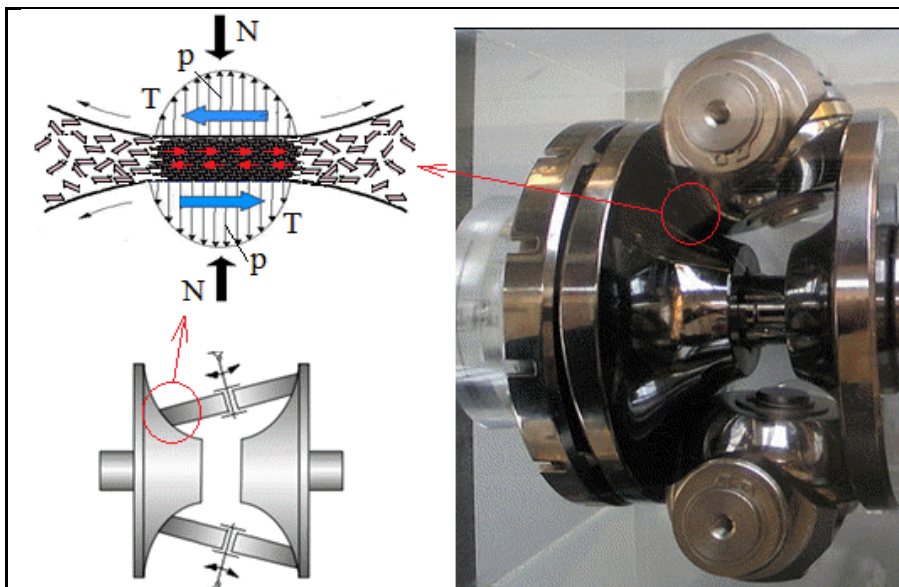
The *main requirements* that the materials used for the active elements of friction drives must meet are: contact strength; wear resistance; a coefficient of friction as high as possible – to avoid high pressing forces – and constant in time.

Pairs of metallic materials characterized by high contact strength and wear resistance:

- hardened steel / hardened steel – for heavily loaded drives, which require high durability and operate with or without lubrication – characterized by minimum overall size and high efficiency; they require high manufacturing and assembly accuracy, together with the reduction of the geometric slip, which could lead to scuffing;
- cast iron / hardened steel – for drives operating with or without lubrication, with the advantage of increased scuffing resistance;
- cast iron / cast iron – for drives operating with lubrication.

Pairs of non-metallic/metallic materials characterized by high coefficients of friction and increased elasticity: textolite, rubber, leather, etc./steel or cast iron – for lightly loaded drives, operating without lubrication, characterized by large overall dimensions and lower efficiency; the non-metallic material is used as linings mounted on the driving element, to ensure uniform wear.

Traction oils for friction drives with variable transmission ratio



Traction oil consists of large molecules with wedge-shaped protrusions which, under extremely high contact pressures, form a structure of orderly aligned molecules and, for a short time, becomes very viscous, behaving almost like a solid body. Unlike transmission oils, which always remain liquid and give low coefficients of friction (0.05...0.06), traction oils, by solidifying in the contact zone, generate high friction forces (coefficients of friction of approx. 0.1) and, in addition, their penetration into the microcracks of the surfaces in contact is reduced. Traction oil must also provide the lubrication and cooling performance of a transmission oil.

Characteristics of traction oils:

- increased coefficient of friction, ensuring increased power transfer;
- they eliminate direct contacts and ensure quiet operation;
- they ensure increased efficiency and dynamic performance;
- they reduce wear by pitting;
- stability at high temperatures;
- lubricating properties as close as possible to those of transmission oils;
- hydraulic behaviour as constant as possible over a wide temperature range (-40...140 °C)

Fig. TFR-T.2.5.1 Scheme of the contact with traction oil

[https://en.wikipedia.org/wiki/Continuously_variable_transmission]

TFR-T.2.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. TFR-T.2.6.1 Forms and causes of improper behavior or removal from service

Forms	Consequences	Occurrence	Causes	Manifestation	Avoidance
Damage of the surfaces of the active elements by pitting	Improper behavior (with irregularities, vibrations and noise)	Lubricated variators	Contact fatigue (pitting)	Microcracks and pits on the active surfaces	Limiting the maximum contact stress to values accepted by the pair of materials
Damage of the surfaces of the active elements by scuffing	Improper behavior (with irregularities, vibrations and noise)	Variators with improper lubrication at high loads and speeds	Scuffing	Increase of friction and of local temperatures; microwelds and scratches on the active surfaces	Limiting the operating speed to permissible values
Damage of the surfaces of the active elements by abrasive wear	Improper behavior (with irregularities, vibrations and noise)	Variators without lubrication and/or unsealed	Abrasive wear	Scratches on the active surfaces	Proper lubrication and sealing

TFR-T.2.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

The parameters characterizing the kineto-statics of a friction drive with variable transmission ratio (variator) are: the angular velocities or speeds, the torques and the powers at the input and, respectively, at the output (fig. TFR-T.2.7.1,a).

Input parameters: angular velocity ω_1 [rad/s] (speed n_1 [rpm]); torque M_{t1} [Nmm]; power P_1 [kW].

Output parameters: angular velocity $\omega_{2x} \in [\omega_{2min}, \omega_{2max}]$ [rad/s] (speed $n_{2x} \in [n_{2min}, n_{2max}]$ [rpm]), variable; torque M_{t2} [Nmm]; power P_2 [kW].

Instantaneous transmission ratio: $i_x = \omega_1/\omega_{2x} = n_1/n_{2x}$ is variable, adjustable by setting certain quantities of the variator, and can take the minimum value $i_{min} = \omega_1/\omega_{2max} = n_1/n_{2max}$ or the maximum value $i_{max} = \omega_1/\omega_{2min} = n_1/n_{2min}$.

The adjustment range is defined as the ratio of the maximum and minimum angular velocities of the driven element of the variator,

$$G = \frac{\omega_{2max}}{\omega_{2min}} = \frac{\omega_{2max}}{\omega_1} \frac{\omega_1}{\omega_{2min}} = \frac{i_{max}}{i_{min}} \quad \text{sau} \quad G = \frac{n_{2max}}{n_{2min}} = \frac{n_{2max}}{n_1} \frac{n_1}{n_{2min}} = \frac{i_{max}}{i_{min}}. \quad (\text{TFR-T.2.7.1})$$

Since the angular velocity of the driven element of variators varies within a certain interval, the parameters characterizing the motion of the driving and driven elements depend on this angular velocity (ω_{2x}) or on the transmission ratio (i_x). Thus, considering the angular velocity of the driving element $\omega_1 = \text{const.}$, the following relations can be written:

$$M_{t1} = \frac{P_1}{\omega_1} = \frac{P_2}{\eta_x \omega_1} = \frac{M_{t2} \omega_{2x}}{\eta_x \omega_1} = \frac{M_{t2}}{\eta_x i_x}, \quad (\text{TFR-T.2.7.2})$$

and, assuming the variation law of one of the quantities (e.g. M_{t1} decreasing non-linearly), the variations in fig. TFR-T.2.7.1,b can be described. In these relations the efficiency η_x was considered dependent on the variable speed and, therefore, on the transmission ratio i_x .

Usually, in the operation of variators ($\omega_1 = \text{const.}$, $\omega_{2x} = \text{variable}$), depending on their purpose, two operating regimes can be encountered, which imply keeping one of the kineto-static parameters of the driven element (M_{t2} or P_2) constant:

- $M_{t2} = \text{const.}$ (a situation frequently encountered in drives used in transfer and conveying systems), which leads to the relations:

$$M_{t1} = \frac{M_{t2}}{\eta_x i_x}, \quad P_1 = \frac{M_{t2} \omega_1}{\eta_x i_x}, \quad P_2 = M_{t2} \omega_{2x} = \frac{M_{t2} \omega_1}{i_x}, \quad (\text{TFR-T.2.7.3})$$

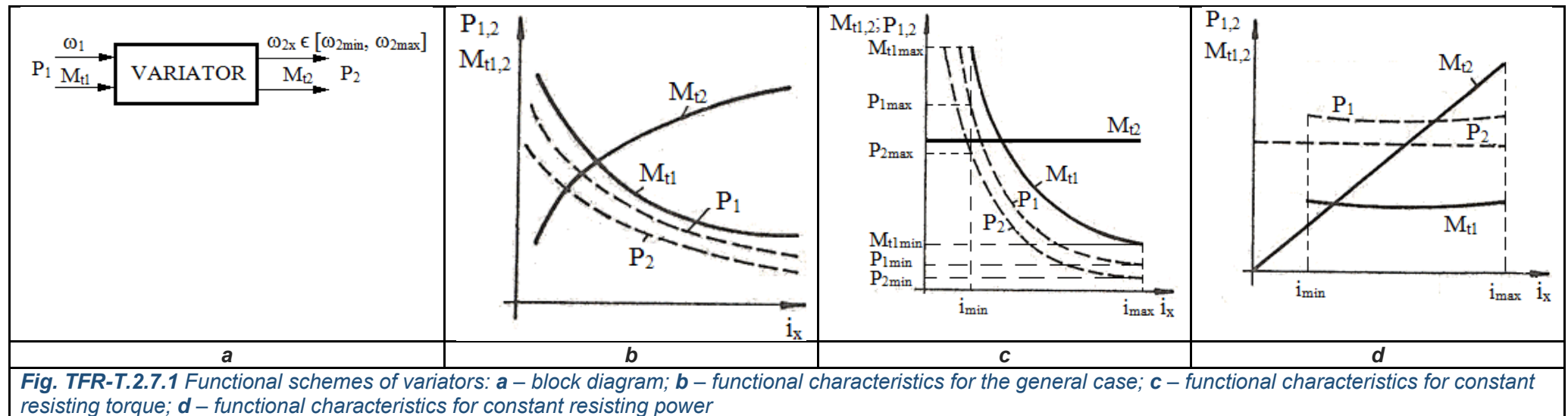
with the shapes in fig. TFR-T.2.7.1,c; in this case the power of the input (driving) motor is not fully used throughout the operation and, in steady state, the limit (maximum and minimum) values are:

$$M_{t1min} = \frac{M_{t2}}{\eta_{max} i_{max}}, \quad M_{t1max} = \frac{M_{t2}}{\eta_{min} i_{min}}, \quad P_{1min} = \frac{M_{t2} \omega_1}{\eta_{max} i_{max}}, \quad P_{1max} = \frac{M_{t2} \omega_1}{\eta_{min} i_{min}}, \quad P_{2min} = \frac{M_{t2} \omega_1}{i_{max}}, \quad P_{2max} = \frac{M_{t2} \omega_1}{i_{min}}. \quad (\text{TFR-T.2.7.4})$$

- $P_2 = \text{const.}$ (a situation requiring full use of the motor power), which leads to the relations:

$$M_{t1} = \frac{P_2}{\omega_1 \eta_x}, \quad P_1 = \frac{P_2}{\eta_x}, \quad M_{t2} = \frac{P_2}{\omega_{2x}} = \frac{P_1}{\omega_1} i_x \eta_x \quad (\text{TFR-T.2.7.5})$$

with the shapes in fig. TFR-T.2.7.1,d; in this case the power of the input (driving) motor varies relatively little with the instantaneous transmission ratio i_x ; for most drives $\eta_x(i_x) = \text{const.}$ can be accepted, so the power of the driving motor can be considered quasi-constant and, consequently, the driving torque (first relation of group (TFR-T.2.7.5)) is also quasi-constant.



TFR-T.2.8 CALCULATION MODELS

TFR-T.2.8.1 Kineto-static calculation models

TFR-T.2.8.1.1 Face variators with rollers

Operation:

Face variator with cylindrical roller, mono (fig. TFR-T.2.8.1.1.1,a): the motion and the torque from the input shaft are transmitted through a spline to the cylindrical roller 1, then by friction to the driven disc 2. The constant pressing force Q is produced by the spring system 4. The system for changing the transmission ratio changes the output speed by changing the contact radius R_{2x} by means of the screw-nut mechanism 5.

Face variator with cylindrical roller, duo (fig. TFR-T.2.8.1.1.1,b): the motion and the torque from the input shaft, integral with the driving disc 3, are transmitted by friction to the cylindrical roller 1, then, also by friction, to the driven disc 2. The constant pressing force Q is produced by the spring system 4. The system for changing the transmission ratio changes the output speed by changing the contact radii R_{1x} and R_{2x} by means of the screw-nut mechanism 5.

Face variator with biconical roller, duo (fig. TFR-T.2.8.1.1.1,c): the motion and the torque from the input shaft are transmitted to the driving disc 1, by friction to the biconical roller 3 and from it, by friction, to the driven disc 2. The constant pressing force Q is produced by the compression spring system. The system for changing the transmission ratio changes the output speed by changing the contact radii R_{1x} and R_{2x} by means of the screw-nut mechanism 4. Window TFR-S.1 presents the constructive solution of a face variator with biconical roller and load-dependent pressing system based on a ball coupling.

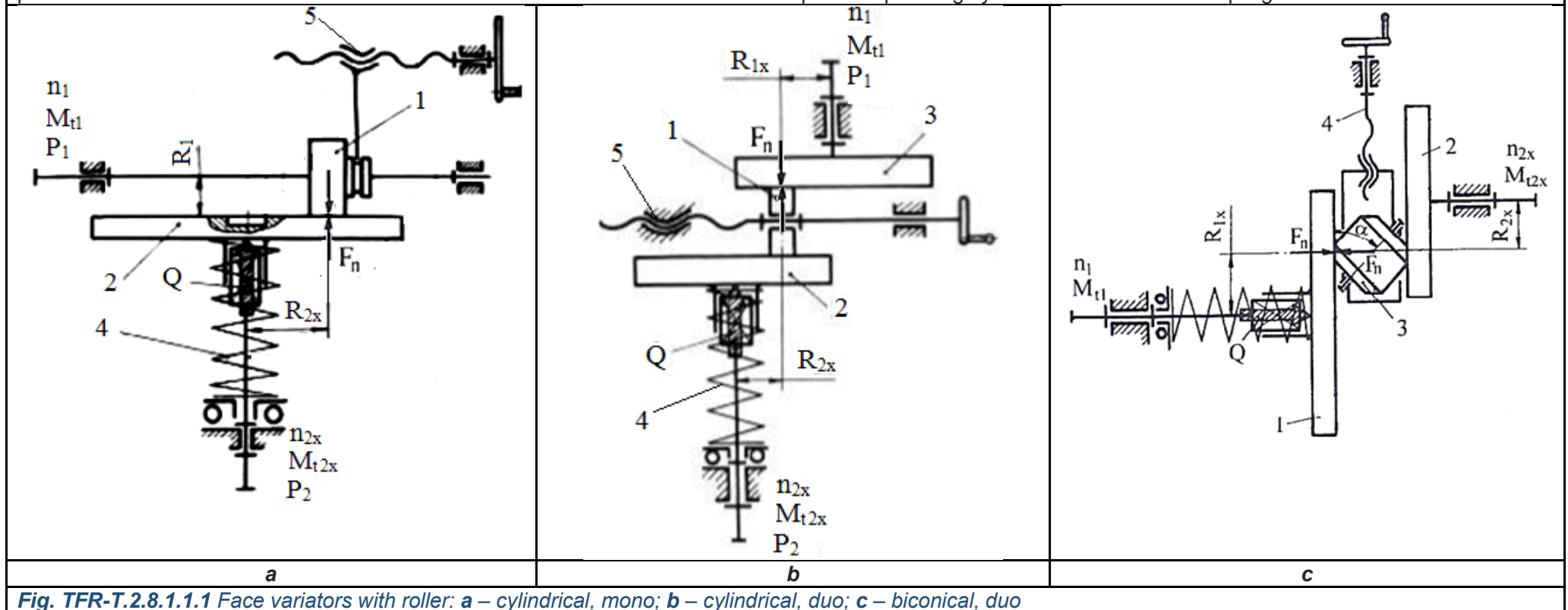


Fig. TFR-T.2.8.1.1.1 Face variators with roller: **a** – cylindrical, mono; **b** – cylindrical, duo; **c** – biconical, duo

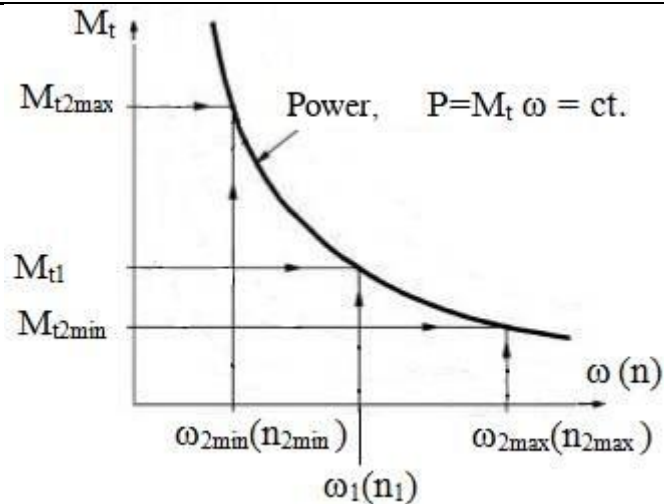


Fig. TFR-T.2.8.1.1.2 Functional characteristic, $M_t - \omega$

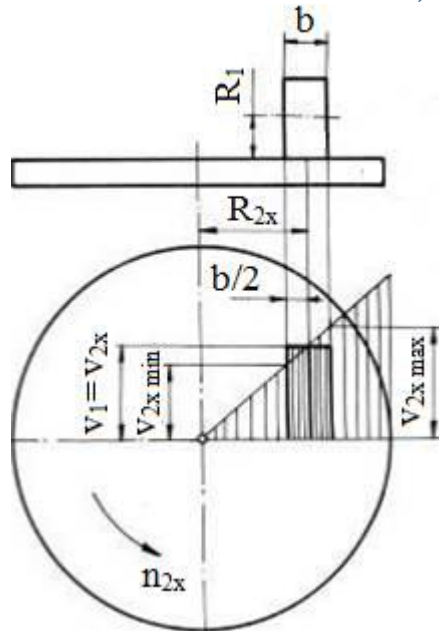


Fig. TFR-T.2.8.1.1.3 Calculation scheme of the geometric slip

Main functional parameters:

- transmission ratios (instantaneous, minimum, maximum): $i_x = n_1/n_{2x} = R_{2x}/R_1$, $i_{\min} = n_1/n_{2\max} = R_{2\min}/R_1$ and, respectively, $i_{\max} = n_1/n_{2\min} = R_{2\max}/R_1$, for mono variators; $i_x = n_1/n_{2x} = R_{2x}/R_{1x}$, $i_{\min} = n_1/n_{2\max} = R_{2\min}/R_{1\max}$ and, respectively, $i_{\max} = n_1/n_{2\min} = R_{2\max}/R_{1\min}$, for duo variators;
- speed adjustment range,

$$G = \frac{n_{2\max}}{n_{2\min}} = \frac{i_{\max}}{i_{\min}} = \frac{\frac{R_{2\max}}{R_{1\min}}}{\frac{R_{2\min}}{R_{1\max}}} = \frac{R_{2\max}}{R_{1\min}} \frac{R_{1\max}}{R_{2\min}}; \quad (\text{TFR-T.2.8.1.1.1})$$

for the mono variant (fig. TFR-T.2.8.1.1.1,a), $R_{1\min} = R_{1\max} = R_1$ is considered in the relations above.

- geometric slip (fig. TFR-T.2.8.1.1.3),

$$\xi_g = \frac{v_{a1\max}}{v_1} = \frac{v_{2\max} - v_1}{v_1} = \frac{\omega_{2x} \left(R_{2x} + \frac{b}{2} \right) - \omega_{2x} R_{2x}}{\omega_{2x} R_{2x}} = \frac{b}{2 R_{2x}}; \quad (\text{TFR-T.2.8.1.1.2})$$

- the functional characteristic in fig. TFR-T.2.8.1.1.2 is defined by the operating case $P_2 = \text{const.}$ (fig. TFR-T.2.7.1,d), which implies full use of the power of the driving motor.

Condition for transmitting the torque, in the case $P_2 = \text{const.}$,

$$\mu F_{n\max} R_{1\min} = c M_{t1} \quad (\text{TFR-T.2.8.1.1.3})$$

where μ is the sliding friction coefficient (corresponding to the materials of the elements in contact),

$F_{n\max}$ – the maximum normal force in the contact at radius $R_{1\min}$ ($R_{1\min} = R_1$, for the mono variant), c

– the safety coefficient against slipping ($c = 1.15 \dots 1.25$).

Pressing force (for the constant-pressure variants),

$$Q \geq F_{n\max} = \frac{c M_{t1}}{\mu R_{1\min}}, \quad (\text{TFR-T.2.8.1.1.4})$$

ensures the friction force needed to drive the driven disc by friction in any position of the contacts.

Note.

The geometric slip can be reduced by using narrow rollers or even rollers with a semicircular profile, for which the theoretical contact is a point, and/or driven discs with a minimum radius as large as possible (although the overall size increases); the use of the biconical roller (fig. TFR-T.2.8.1.1.1,c) substantially reduces the geometric slip because of the short length of the contact line

TFR-T.2.8.1.2 Spherical variators

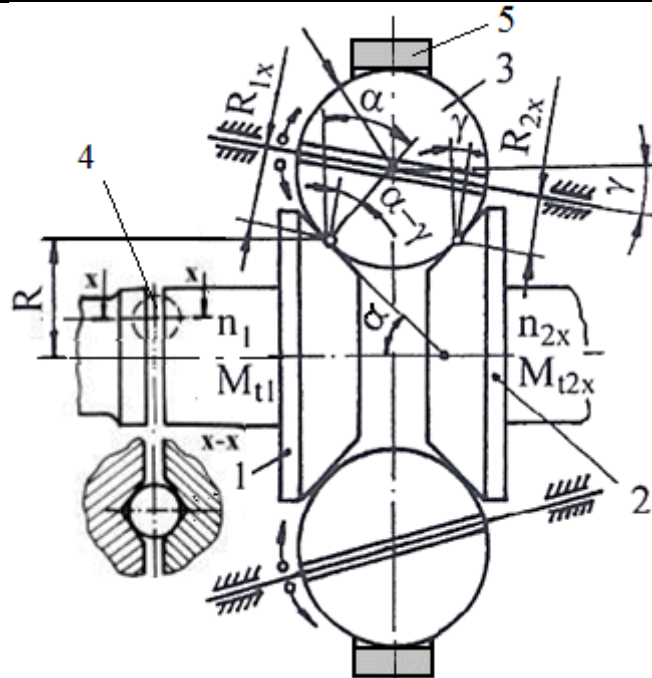


Fig. TFR-T.2.8.1.2.1 Calculation scheme of the spherical variator

Operation:

The torque M_{t1} is transmitted from the input shaft through the pressing system 4, through the conical disc 1, by friction to the spherical bodies 3 (evenly spaced) and from them, also by friction, to the driven conical disc 2. The transmission ratio (the two contact radii) is changed by the synchronous rotation of the support arms 5. The spherical bodies 3 are kept in contact by the rotating ring 5. The pressing force needed to transmit the load by friction is produced by the pressing system 4, which pushes disc 1 axially with a force depending on the transmitted torque. Window TFR-S.2 presents a constructive solution of a spherical variator.

Main functional parameters:

- transmission ratios (instantaneous, minimum, maximum):

$$i_x = \frac{R_{2x}}{R_{1x}} = \frac{R_3 \cos(\alpha \mp \gamma)}{R_3 \cos(\alpha \pm \gamma)} = \frac{\cos(\alpha \mp \gamma)}{\cos(\alpha \pm \gamma)}, \quad (\text{TFR-T.2.8.1.2.1})$$

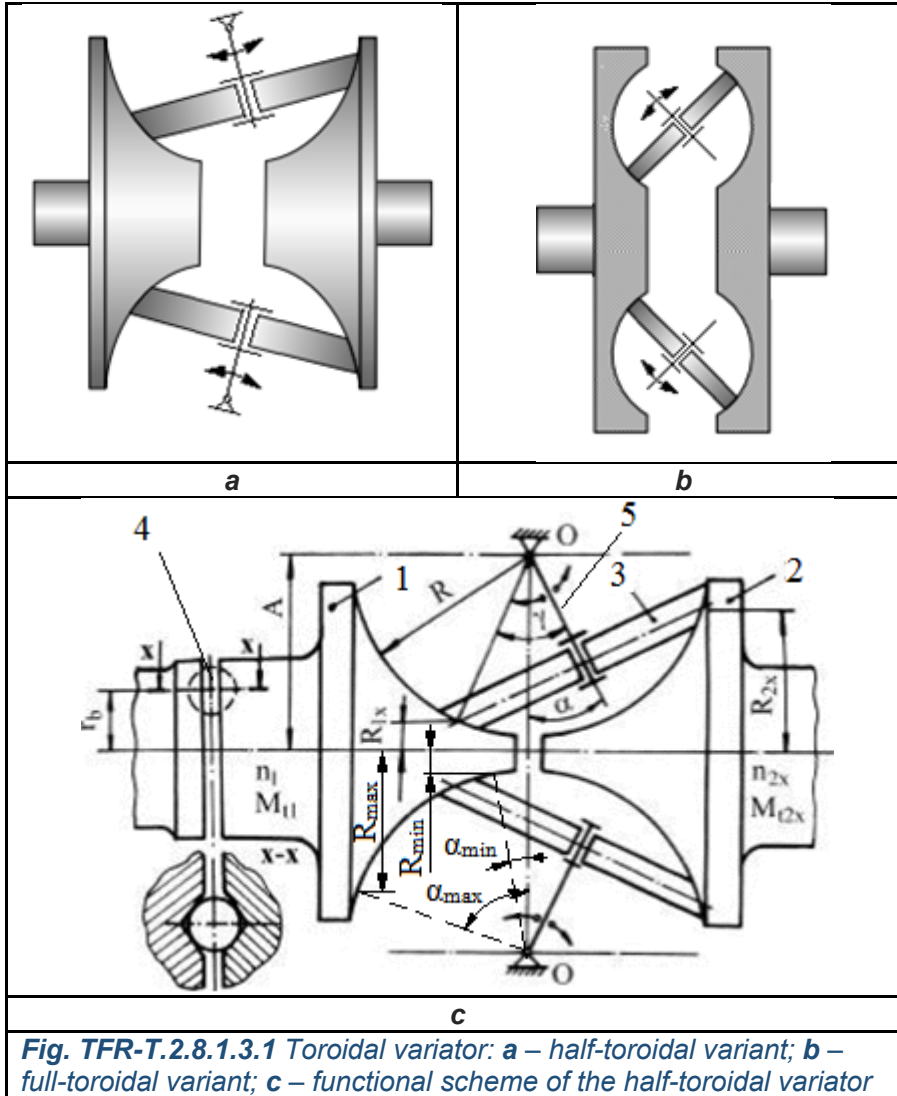
$i_{\min} = n_1/n_{2\min} = R_{1\min}/R_{2\max}$ and, respectively, $i_{\max} = n_1/n_{2\max} = R_{1\max}/R_{2\min}$, where R_{1x} , R_{2x} are the radii of the contacts of the spherical element 3 with disc 1 and, respectively, with disc 2; R_3 – the radius of sphere 3; γ – the inclination angle of the sphere axis (for $\gamma = 0$ the sphere axis is parallel to the disc axes and $i_x = 1$); α – the half-angle of the cones ($\alpha = 40^\circ \dots 50^\circ$); in this relation the upper signs are used when the roller axes are inclined to the left (the variator is a reducer) and the lower signs when they are inclined to the right (fig. TFR-T.2.8.1.2.1, the variator is a multiplier).

- speed adjustment range,

$$G = \frac{n_{2\max}}{n_{2\min}} = \frac{n_{2\max}/n_1}{n_{2\min}/n_1} = \frac{i_{\max}}{i_{\min}} = \frac{R_{1\max}}{R_{1\min}} \frac{R_{2\max}}{R_{2\min}} \quad (\text{TFR-T.2.8.1.2.2})$$

where $R_{\max}$, $R_{\min}$ are the maximum and minimum contact radii (R_{1x} , R_{2x}).

TFR-T.2.8.1.3 Toroidal variators



Depending on the shapes of the toroidal surfaces, two variants can be defined: half-toroidal variators (fig. TFR-T.2.8.1.3.1,a) and full-toroidal variators (fig. TFR-T.2.8.1.3.1,b), which achieve larger adjustment ranges.

Operation:

The torque M_{t1} is transmitted from the input shaft through the pressing system 4, through the toroidal disc 1, by friction to the rollers 3 and from them, also by friction, to the driven toroidal disc 2. The transmission ratio (the two contact radii) is changed by the synchronous rotation of the support arms 5. The pressing force needed to transmit the load by friction is produced by the pressing system 4, which pushes disc 1 axially with a force depending on the transmitted torque. Window TFR-S.3 presents constructive solutions of variators based on the two variants.

Main functional parameters:

- transmission ratios (instantaneous, minimum, maximum):

$$i_x = \frac{R_{2x}}{R_{1x}} = \frac{A - R \cos(\gamma \pm \alpha)}{A - R \sin(\frac{\pi}{2} - (\gamma \mp \alpha))} = \frac{A - R \cos(\gamma \pm \alpha)}{A - R \cos(\gamma \mp \alpha)}, \quad (\text{TFR-T.2.8.1.3.1})$$

$$i_{\min} = n_1/n_{2\max} = R_{\min}/R_{\max} \text{ and, respectively, } i_{\max} = n_1/n_{2\min} = R_{\max}/R_{\min},$$

where R_{1x} , R_{2x} are the radii of the contacts between roller 3 and disc 1 and, respectively, disc 2; R – the radius of curvature of the active zones of discs 1 and 2; γ – the half-angle of the roller (constant); $\alpha_{\min} \leq \alpha \leq \alpha_{\max}$ – the angles of the positions of the roller axes; in this relation the upper signs are used when the roller axes are inclined to the left (the variator is a multiplier) and the lower signs when they are inclined to the right (fig. TFR-T.2.8.1.3.1,c, the variator is a reducer).

- speed adjustment range,

$$G = \frac{n_{2\max}}{n_{2\min}} = \frac{n_{2\max}/n_1}{n_{2\min}/n_1} = \frac{i_{\max}}{i_{\min}} = \left(\frac{R_{\max}}{R_{\min}} \right)^2 \quad (\text{TFR-T.2.8.1.3.2})$$

where $R_{\max}$, $R_{\min}$ are the maximum and minimum contact radii.

Fig. TFR-T.2.8.1.3.1 Toroidal variator: **a** – half-toroidal variant; **b** – full-toroidal variant; **c** – functional scheme of the half-toroidal variator

TFR-T.2.8.1.4 Planetary variators with conical rollers

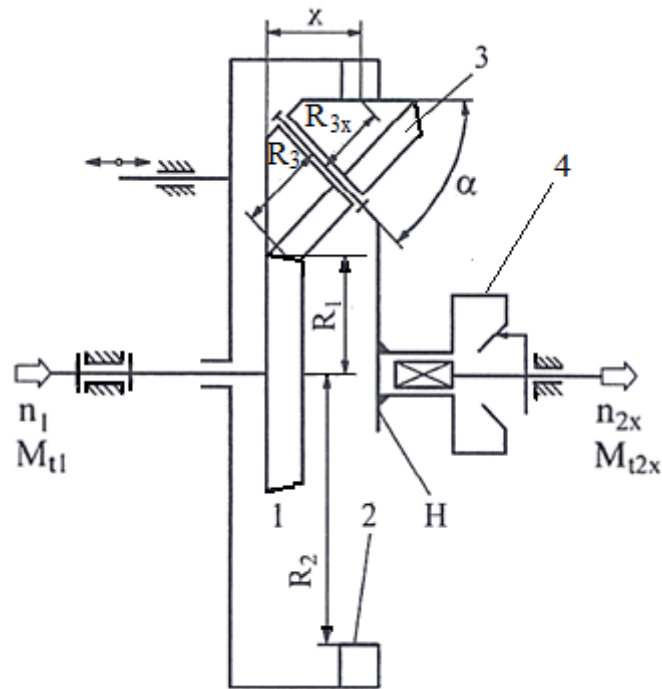


Fig. TFR-T.2.8.1.4.1 Functional scheme of the planetary variator

Operation:

The torque M_{t1} is transmitted from the conical disc 1 by friction to the rollers 3 (evenly spaced) and from them, through the planet carrier arms H and the pressing system 4, to the output shaft. The transmission ratio is changed by the axial displacement of ring 2. The pressing force needed to transmit the load by friction is produced by the pressing system 4, which pushes the arms H and the rollers 3 axially with a force depending on the transmitted torque. Window TFR-S.4 presents a constructive solution of a planetary variator.

Main functional parameters:

- transmission ratios (instantaneous, minimum, maximum):

$$i_x = i_{1H}^2 = 1 - i_{12}^H = 1 - \frac{R_3}{R_1} \frac{R_2}{R_{3x}} = 1 - \frac{R_3}{R_1} \frac{R_2}{x \sin \alpha}, \quad (\text{TFR-T.2.8.1.4.1})$$

$i_{\min} = n_1/n_{2\max} = 1 - (R_3/R_1)/(R_2/R_{3\min})$ and, respectively, $i_{\max} = n_1/n_{2\min} = 1 - (R_3/R_1)/(R_2/R_{3\max})$, where R_{3x} is the radius of roller 3 in contact with ring 2; R_2 – the radius of ring 2; R_3 – the radius of roller 3 in contact with disc 1 (constant); R_1 – the radius of disc 1 in contact with roller 3 (constant); α

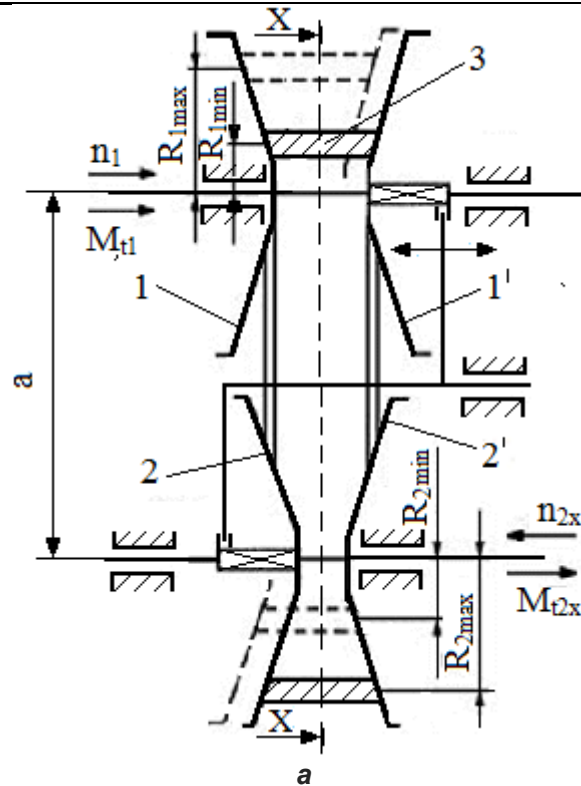
– the inclination angle of the conical roller 3; x – the position of ring 2 relative to roller 3; i_{1H}^2 – the transmission ratio from disc 1 to the planet carrier 4 with ring 2 fixed (friction drive with moving axis);

i_{12}^H – the transmission ratio from disc 1 to ring 2 with the planet carrier 4 fixed (friction drive with fixed axis, obtained by inverting the motion); $R_{3\max, \min}$ – the maximum and, respectively, minimum radius of roller 3 in contact with ring 2;

- speed adjustment range,

$$G = \frac{i_{\max}}{i_{\min}} = \frac{1 - (R_3/R_1)/(R_2/R_{3\max})}{1 - (R_3/R_1)/(R_2/R_{3\min})} \quad (\text{TFR-T.2.8.1.4.2})$$

TFR-T.2.8.1.5 Variators with movable cones and rigid element (ring)



Operation:

The torque M_{t1} is transmitted from the conical discs 1, 1' by friction to the rigid ring 3 and from it, also by friction, through the conical discs 2, 2' to the output shaft. The transmission ratio is changed by the simultaneous axial displacement of discs 2, 2'. The pressing force needed to transmit the load by friction is produced automatically once the self-wedging conditions of the ring in the gaps between the driving cones and, respectively, the driven cones are met. Self-wedging of the rigid element (ring) 3 is a consequence of the wedge effect resulting from the (small) wedging angle β between the radial forces ($N_1 \cos \alpha$) and the resultant forces R (fig. TFR-T.2.8.1.5.1,b); therefore, the pressure angle (the complement of the wedging angle β) has large values and optimal conditions are achieved for transmitting the force in the contact zones between the free ring 3 and the conical discs 1, 1', 2, 2'. The own weight of the rigid element (ring) 3 is the initial pressing force needed to start transmitting the load at start-up. Window TFR-S.5 presents a constructive solution of a variator with movable cones and rigid element (ring).

Main functional parameters:

- transmission ratios (instantaneous, minimum, maximum):

$i_x = n_1/n_{2x} = R_{2x}/R_{1x}$, $i_{min} = n_1/n_{2max} = R_{2min}/R_{1max}$ and, respectively, $i_{max} = n_1/n_{2min} = R_{2max}/R_{1min}$;

- speed adjustment range,

$$G = \frac{n_{2max}}{n_{2min}} = \frac{n_{2max}/n_1}{n_{2min}/n_1} = \frac{i_{max}}{i_{min}} = \frac{R_{1max}}{R_{1min}} \frac{R_{2max}}{R_{2min}}, \quad (\text{TFR-T.2.8.1.5.1})$$

where R_{1max} , R_{1min} , R_{2max} , R_{2min} are the maximum and minimum contact radii.

Condition for transmitting the load by friction:

$c F_t = 2 \mu N_1$, (TFR-T.2.8.1.5.2)

where c is the safety coefficient against slipping; $F_t = M_{t1}/R_{1min}$ – the maximum useful tangential force (transmitted by friction); μ – the sliding friction coefficient of the materials of the rigid ring and of the conical discs; N_1 – the normal force in the active contacts.

Wedging (automatic pressing) condition:

$F_t = 2 N_1 \cos \alpha \tan \beta$ (TFR-T.2.8.1.5.3)

where $F_t = M_{t1}/R_{1max}$ is the minimum useful tangential force (transmitted by friction); N_1 – the normal force in the contacts; α – the half-angle of the cones; β – the wedging angle.

Note.

From relations (TFR-T.2.8.1.5.2) and (TFR-T.2.8.1.5.3) the condition results,

$$\frac{N_1}{F_t} = \frac{c}{2\mu} = \frac{1}{2 \cos \alpha \tan \beta} = ct. \quad (\text{TFR-T.2.8.1.5.4})$$

which shows that the safety coefficient against slipping is independent of the transmission ratio; this relation can be used to determine the maximum wedging angle β which ensures wedging and the transmission of the load.

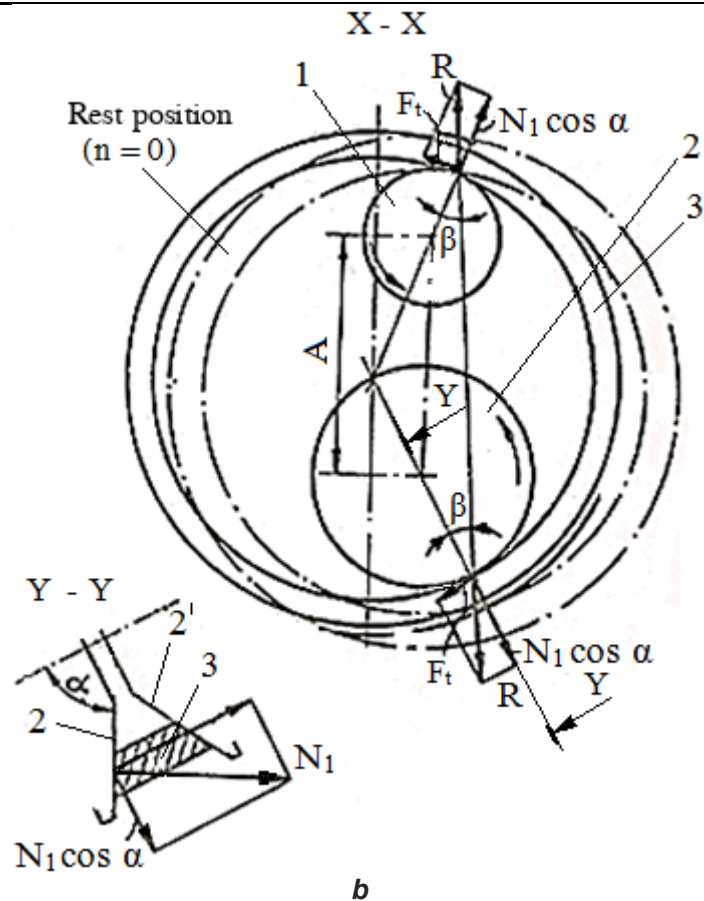
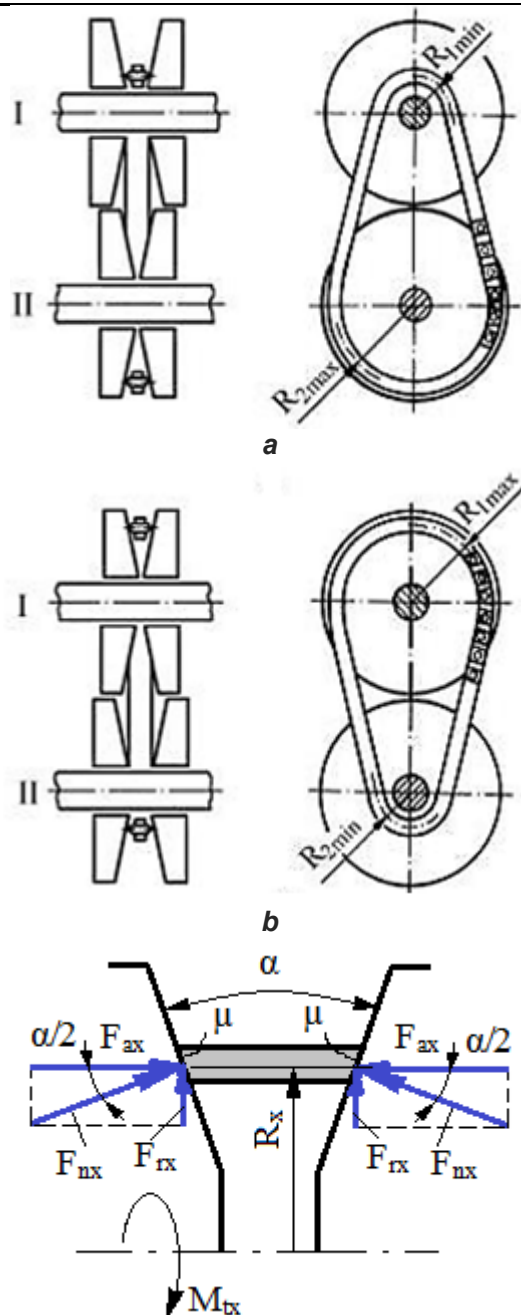


Fig. TFR-T.2.8.1.5.1 Variator with movable cones and rigid element (ring): **a** – axial scheme; **b** – scheme of the self-wedging process

TFR-T.2.8.1.6 Variators with movable cones and flexible element of the (non-metallic) belt type or articulated element (chain)



Operation:

Variators with movable cones consist of two pairs of cones mounted on the input shaft I (fig. TFR-T.2.8.1.6.1) and on the output shaft II. To change the transmission ratio, the position of the two cones can be adjusted by keeping one cone fixed and displacing the other axially, or by the simultaneous axial displacement of both cones.

Main functional parameters:

- transmission ratios (instantaneous, minimum, maximum):

$i_x = n_1/n_{2x} = R_{2x}/R_{1x}$, $i_{\min} = n_1/n_{2\max} = R_{2\min}/R_{1\max}$ and, respectively, $i_{\max} = n_1/n_{2\min} = R_{2\max}/R_{1\min}$, where R_{1x} , R_{2x} are the contact radii.

- speed adjustment range,

$$G = \frac{n_{2\max}}{n_{2\min}} = \frac{n_{2\max}/n_1}{n_{2\min}/n_1} = \frac{i_{\max}}{i_{\min}} = \frac{R_{1\max}}{R_{1\min}} \frac{R_{2\max}}{R_{2\min}}, \quad (\text{TFR-T.2.8.1.6.1.1})$$

where $R_{1\max}$, $R_{1\min}$, $R_{2\max}$, $R_{2\min}$ are the maximum and minimum contact radii.

Condition for transmitting the load by friction in an arbitrary position of the flexible element:

$$2\mu F_{nx} R_x = c M_{tx}, \quad (\text{TFR-T.2.8.1.6.1.2})$$

where c is the safety coefficient against slipping; F_{nx} – the normal pressing force; μ – the sliding friction coefficient of the materials of the flexible element and of the conical discs; M_{tx} – the transmitted torque; R_x – the contact radius.

From relation (TFR-T.2.8.1.6.1.2), taking into account fig. TFR-T.2.8.1.6.1, the axial pressing force of the discs results,

$$F_{ax} = \frac{c M_{tx}}{2 \mu R_x} \cos \frac{\alpha}{2}$$

and the radial force loading the shaft,

$$F_{rx} = \frac{c M_{tx}}{2 \mu R_x} \sin \frac{\alpha}{2}$$

c

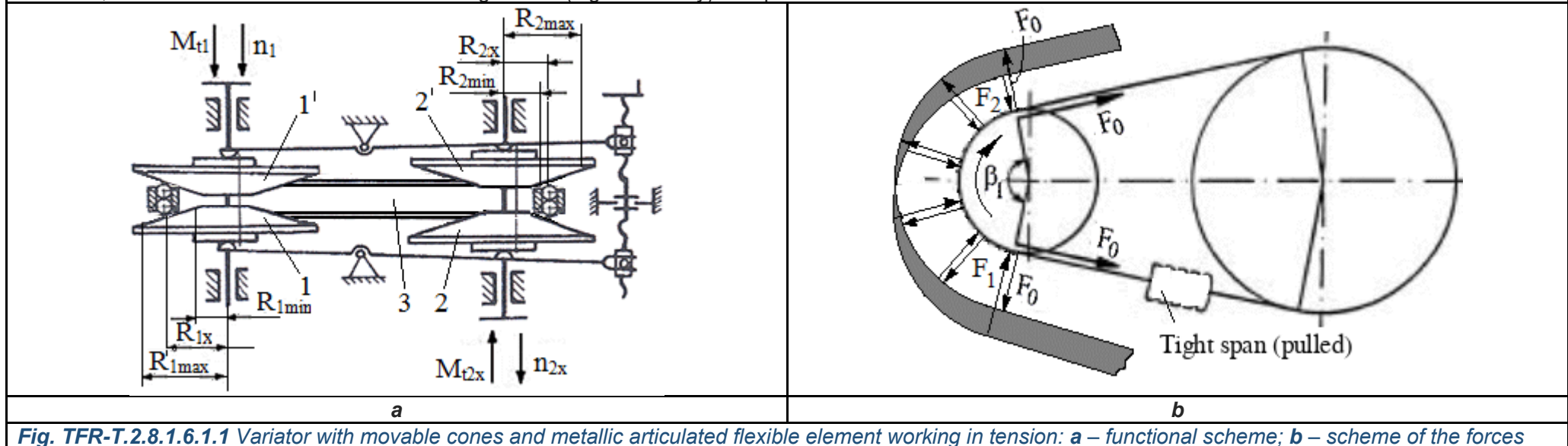
Fig. TFR-T.2.8.1.6.1 Functional schemes: **a** – multiplying; **b** – reducing; **c** – of the forces

TFR-T.2.8.1.6.1 Variators with movable cones and flexible (non-metallic belt) or articulated (chain) element working in tension

Operation:

The torque M_{t1} is transmitted from the conical discs 1, 1' by friction to the flexible element (chain) 3 and from it, also by friction, through the conical discs 2, 2' to the output shaft. The transmission ratio is changed by the simultaneous axial displacement of discs 1, 1' and 2, 2'. Window TFR-S.6 presents a constructive solution of a variator with a flexible belt-type or articulated chain-type element working in tension. These variators are characterized by high forces (higher than the pretensioning force, fig. TFR-T.2.8.1.6.1.1) in the tight span and low forces in the slack span (lower than the pretensioning force).

Metallic articulated elements of the chain type, frequently used in practice, can have various configurations. Fig. TFR-T.2.8.1.6.1.2 shows three constructive solutions, which differ in the extent to which rolling friction (high efficiency) is implemented.



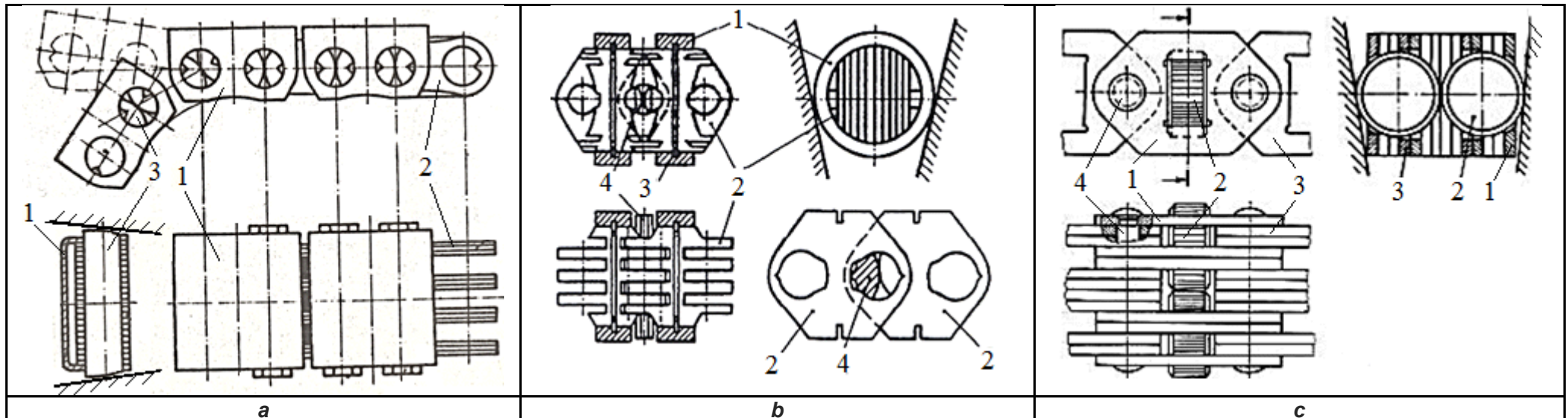
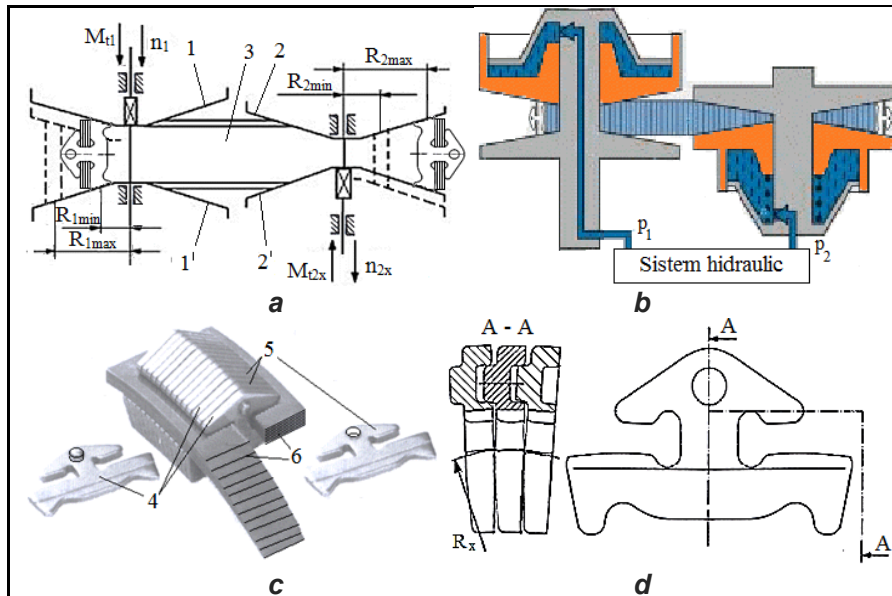


Fig. TFR-T.2.8.1.6.1.2 Metallic articulated elements (chains): **a** – chains with pins and cut-out link plates (1 – outer link plate, 2 – inner link plate, 3 – pin); **b** – chains with rings (1 – ring, 2 – link plate, 3 – elastic ring, 4 – pin); **c** – roller chains (1 – outer link plate, 2 – roller, 3 – inner link plate, 4 – pin)

TFR-T.2.8.1.6.2 Variators with movable cones and metallic flexible element working in compression (Van Doorne)



Operation:

The torque M_{t1} is transmitted from the conical discs 1, 1' by friction to the pushing flexible element 3 and from it, also by friction, through the conical discs 2, 2' to the output shaft. The transmission ratio is changed by the simultaneous axial displacement of discs 1 and 2' by hydraulic motors. The force is transmitted through the flexible element 3 by the direct contact (shape) between the segments 4 and 5 (fig. TFR-T.2.8.1.6.2.1,c,d), supported by the two sets of metal bands 6.

Main functional parameters:

- transmission ratios (instantaneous, minimum, maximum):

$$i_x = n_1/n_{2x} = R_{2x}/R_{1x}, i_{\min} = n_1/n_{2\max} = R_{2\min}/R_{1\max} \text{ and, respectively, } i_{\max} = n_1/n_{2\min} = R_{2\max}/R_{1\min},$$

where R_{1x} , R_{2x} are the contact radii;

- speed adjustment range,

$$G = \frac{n_{2\max}}{n_{2\min}} = \frac{n_{2\max}/n_1}{n_{2\min}/n_1} = \frac{i_{\max}}{i_{\min}} = \frac{R_{1\max}}{R_{1\min}} \frac{R_{2\max}}{R_{2\min}} \quad (\text{TFR-T.2.8.1.6.2.1})$$

where $R_{1\max}$, $R_{1\min}$, $R_{2\max}$, $R_{2\min}$ are the maximum and minimum contact radii.

- static forces:

The torque at the driving shaft, transmitted through the active (compressed) span, is determined with the relation,

$$M_{t1} = (F_1 - F_2) R_{1\min} \quad (\text{TFR-T.2.8.1.6.2.2})$$

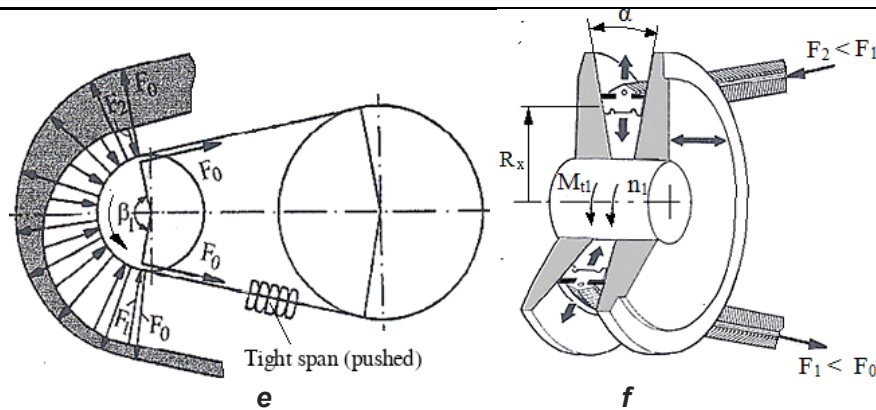


Fig. TFR-T.2.8.1.6.2.1 Variator with movable cones and metallic flexible element working in compression: **a** – functional scheme; **b** – scheme of the system for changing the transmission ratio and of the pressing system; **c** – constructive structure of the pushing flexible element (4 – segment with protrusion, 5 – segment with recess, 6 – circular metal bands); **d** – assembly of the segments; **e, f** – schemes of the forces

where F_1 is the force in the active (compressed) span and F_2 – the force in the passive span; for proper operation the active (pushed) span must remain in tension, $F_1 < F_0$, and $F_1 = K_s F_0$ is considered, with $K_s = 1.2 \dots 1.4$ – safety factor and F_0 – the initial tensioning force of the flexible element.

From Euler's relation (see the chapter Belt drives), applied to this case,

$$\frac{F_0 - F_2}{F_0 - F_1} = e^{\mu\beta_1} \quad (\text{TFR-T.2.8.1.6.2.3})$$

taking into account relation (TFR-T.2.8.1.6.2.2), the maximum pushing force is obtained,

$$F_1 = \frac{1}{(e^{\mu\beta_1} - 1)(K_s - 1)} \frac{M_{t1}}{R_{1\min}}, \quad (\text{TFR-T.2.8.1.6.2.4})$$

and the initial tensioning force,

$$F_0 = \frac{K_s}{(e^{\mu\beta_1} - 1)(K_s - 1)} \frac{M_{t1}}{R_{1\min}}, \quad (\text{TFR-T.2.8.1.6.2.5})$$

where $\mu = \mu_r / \cos(\alpha/2)$ is the reduced coefficient of friction; μ_r – the coefficient of friction of the pair of materials in contact; α – the angle of the segments of the flexible element; β_1 – the wrap angle of the flexible element on the driving discs.

Note. Ensuring a very high pressing force to maintain a firm contact between the friction elements leads to higher power losses in the metal belt and in the bearings. Thus, at all operating points of the variators there is a conflict between the need to ensure a contact pressure sufficient to transmit the load by friction, on the one hand, and the need to minimize the power losses affecting the efficiency and the contact stresses affecting the durability, on the other hand. Window TFR-S.7 presents a constructive solution using the Van Doorne variator in automotive transmissions.

TFR-T.2.8.2 Contact stress calculation model

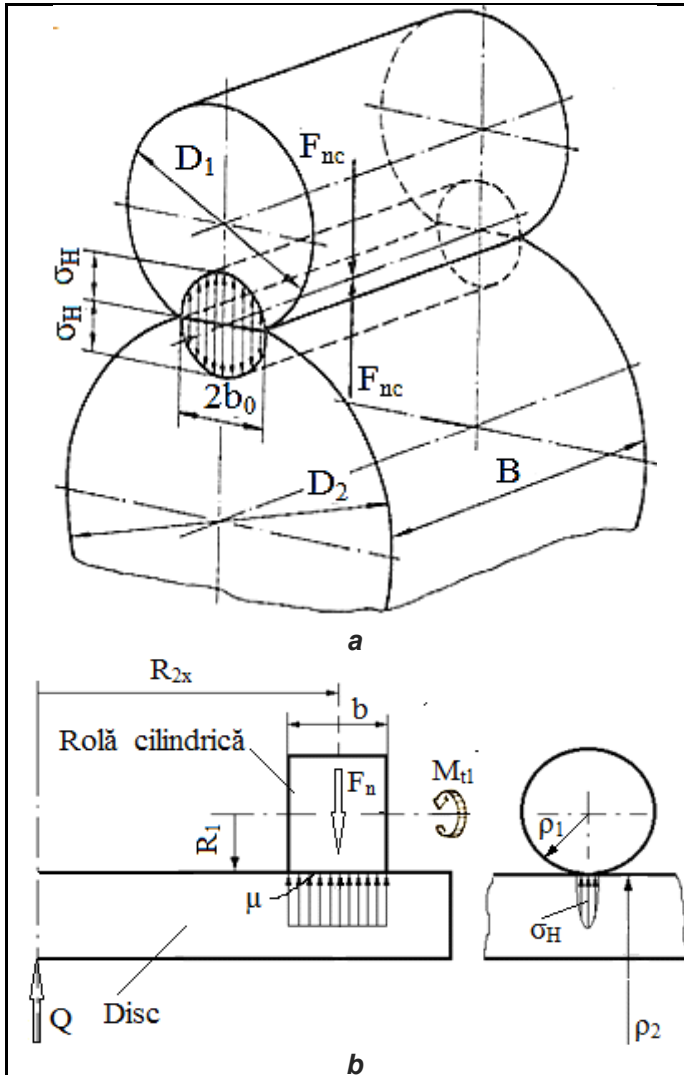


Fig. TFR-T.2.8.2.1 Contact calculation scheme: **a** – Hertz model; **b** – contact calculation model of the face variator with cylindrical roller

For friction variators operating with lubrication, the main form of damage is contact fatigue (pitting) and, to avoid it, the contact stress calculation is necessary.

Reference calculation model for line contact stress [Hertz, 1881] (fig. TFR-T.2.8.2.1,a)

In the contact zone of two cylinders with diameters $D_{1,2}$ and lengths B , loaded with the force F_{nc} , maximum normal contact stresses occur, which, according to Hertz, are determined with the relation,

$$\sigma_H = Z_E \sqrt{\frac{F_{nc}}{l_k}} \frac{1}{\rho} \leq \sigma_{HP}, \quad (\text{TFR-T.2.8.2.1})$$

where $l_k = B$ is the length of the contact line;

$$\frac{1}{\rho} = \frac{1}{\rho_1} \pm \frac{1}{\rho_2} = \frac{1}{D_1/2} \pm \frac{1}{D_2/2},$$

the reduced curvature ($\rho_{1,2}$ are the radii of curvature of the two cylinders; the sign (+) corresponds to external contact (fig. AEC-T.1,a) and the sign (-) to internal contact);

$$Z_E = \sqrt{\frac{1}{\pi \left(\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)}}$$

the elasticity factor, where $E_{1,2}$ and $v_{1,2}$ are the moduli of elasticity and, respectively, the transverse contraction (Poisson) coefficients of the materials of the two cylinders, and

$$\sigma_{HP} = \frac{\sigma_{Hlim}}{S_{Hmin}},$$

the permissible contact stress, where σ_{Hlim} is the limit contact stress and S_{Hmin} – the minimum safety coefficient against contact fatigue of the material.

Assumptions of the Hertz model: the materials of the bodies are homogeneous, isotropic and elastic according to Hooke's law; the normal force is applied statically; the contact stresses are uniformly distributed along the contact length; the surfaces of the cylinders are perfectly smooth; friction forces are not taken into account.

Contact stress calculation of friction drives with variable transmission ratio

Particularization of the Hertz model parameters for the contact of the face variator (fig. TFR-T.2.8.2.1,b; TFR-T.2.8.1.1.1,a):

$\rho_1 = R_1$, $\rho_2 = \infty$, $l_k = b$,

$$F_{nc} = Q = c F_n = c \frac{M_{t2}}{\mu R_{2min}}, \quad (\text{TFR-T.2.8.2.2})$$

for the case of constant resisting torque, $M_{t2} = \text{const.}$ (see fig. TFR-T.2.7.1,c), or

$$F_{nc} = Q = c F_n = c \frac{M_{t1}}{\mu R_1} = c \frac{P_2}{\omega_1 \mu R_1 \eta_{min}}, \quad (\text{TFR-T.2.8.2.3})$$

	for the case of constant resisting power, $P_2 = \text{const.}$ (see fig. TFR-T.2.7.1,d; rel. (TFR-T.2.7.5)). In these relations, μ is the sliding friction coefficient between the surfaces in contact, c – the safety coefficient against slipping, $\eta_{\min}$ – the minimum efficiency.
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TSP. SCREW-NUT DRIVES

TSP-T. CONSTRUCTION AND MODELING OF SCREW-NUT DRIVES

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TSP-T.2 CONSTRUCTIVE STRUCTURE 8-3

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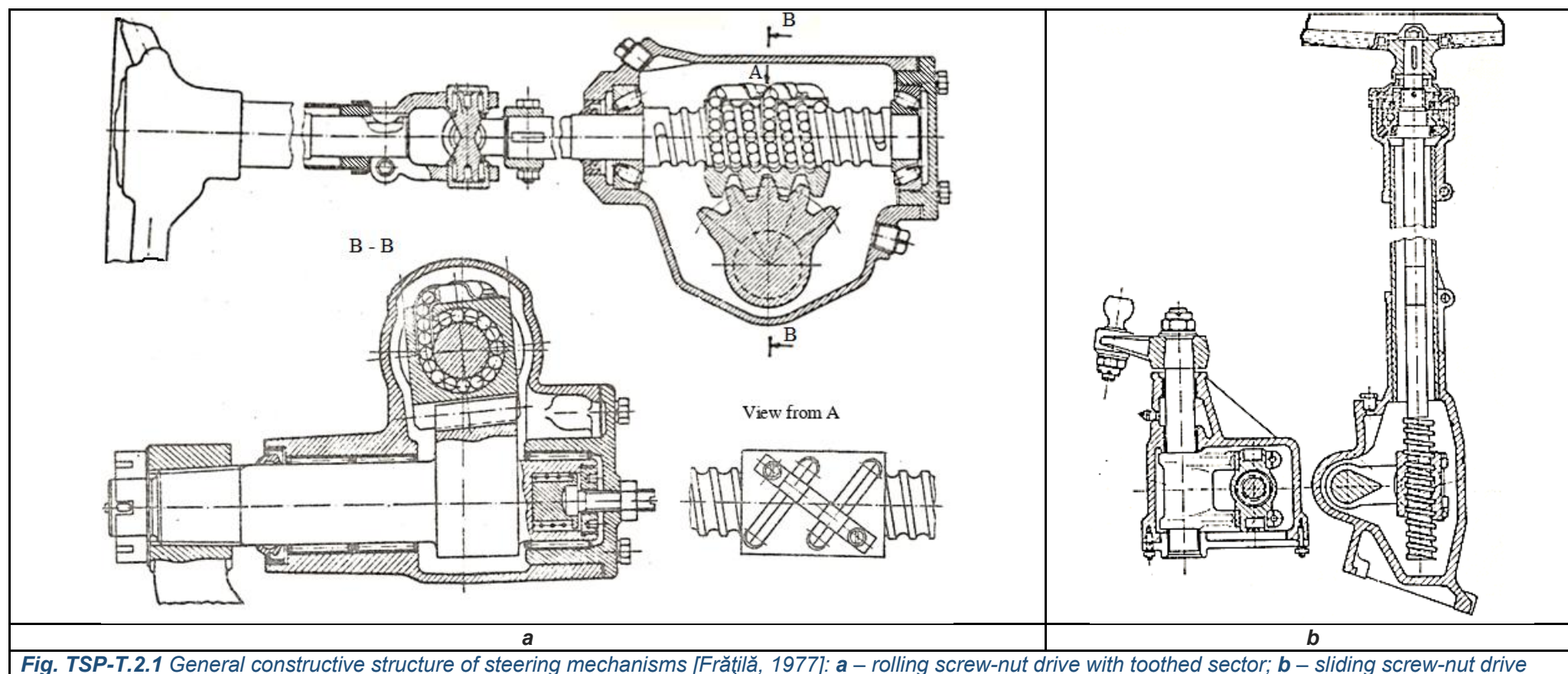
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TSP-T.1 DEFINITION

Screw-nut drives are based on mechanisms with a helical pair (fig. TSP-T.7.1) of constant lead (materialized by a sliding or a rolling thread), used to transform rotational motion into translational motion and/or, under certain constructive conditions (when the self-locking condition is not met), translational motion into rotational motion (fig. TSP-T.7.1,e,f,g,h).

TSP-T.2 CONSTRUCTIVE STRUCTURE



TSP-T.3 AREAS OF USE, ADVANTAGES AND DISADVANTAGES

Main areas of use: steering actuation mechanisms of heavy trucks; lifting devices (jacks); machine tools.

Tab. TSP-T.3.1 *Advantages and disadvantages**

<i>Advantages</i>	<i>Disadvantages</i>
- simple constructive structures; - the elements of sliding pairs are relatively simple to manufacture, using technologies that do not require special conditions or equipment; - quiet operation, especially for constructions with sliding friction; - high efficiency, especially for constructions with rolling friction; - easy maintenance in operation	- low kinematic accuracy, especially for constructions with sliding friction; - clearances appearing in time, caused by the wear of the surfaces in contact; - low efficiency, especially for constructions with sliding friction; - noise and vibrations in operation, especially for constructions with rolling friction
* Compared to other types of drives transforming rotation into translation (rack and pinion, lever mechanisms, etc.)	

TSP-T.4 CLASSIFICATION

Tab. TSP-T.4.1 *Classification of screw-nut drives*

<i>Criterion</i>	<i>Type</i>	<i>Examples</i>
Type of thread	Trapezoidal	Fig. TSP-T.8.1.2.1,c
	Square	Fig. TSP-T.8.1.2.1,b
Type of friction	Sliding (mixed, fluid)	Fig. TSP-T.2.1,b; TSP-T.8.1.2.1,d
	Rolling	Fig. TSP-T.2.1,a; TSP-T.8.1.1.1,b
Type of transmission ratio	With constant transmission ratio	
	With variable transmission ratio	

TSP-T.5 MATERIALS AND TECHNOLOGIES

Material pairs: steel-steel; steel-bronze; steel-antifriction cast iron.

TSP-T.6 FORMS AND CAUSES OF REMOVAL FROM SERVICE OR IMPROPER BEHAVIOR

Tab. TSP-T.6.1 Forms and causes of removal from service or improper behavior of screw-nut drives

Failure type	Forms	Occurrence	Causes	Manifestation	Avoidance
Improper behavior (with irregularities, vibrations and noise) of rolling screw-nut drives	Damage of the raceways and/or rolling elements by contact fatigue (pitting)	Mainly for elements made of steels with low surface hardness (below 45 HRC)	Fatigue of the surface layers of the raceways and/or rolling elements, caused by contact stresses varying in time	Initial microcracks on the active surfaces, as a consequence of the hydrostatic pressure of the oil, followed by the detachment of small particles of material, resulting in small pits. The development of the pits in time leads to improper operation, with vibrations and noise	Limiting by calculation the maximum (fatigue) contact stresses to permissible stresses. Heat and thermochemical treatments that increase the surface contact strength. Reducing the roughness of the active surfaces. Using lubricants with additives.
	Damage of the raceways and/or rolling elements by scuffing	In heavily loaded drives with high speeds and improper lubrication	Increased slip between the active surfaces; high roughness of the flanks	Scratches generated by the formation and breaking of microwelds, caused by high local loads and high temperatures	Improving the lubrication conditions (lubricants with additives); increasing the manufacturing and assembly accuracy; reducing the roughness of the active surfaces
	Damage of the flanks by abrasive wear	In open (unprotected) drives with improper lubrication	Increased slip between the active surfaces; high roughness of the flanks	Removal of fine particles of material from the raceways and/or rolling elements by abrasive particles (dust and impurities) in the contact zone	Proper lubrication, sealing and protection
	Damage of the surfaces of the raceways and/or rolling elements by plastic flow	In drives whose active elements are made of steels with low hardness	Overloads, large slip and poor lubrication	Dents on the raceways and/or rolling elements, as a result of exceeding the yield stresses	Reducing the contact loads and increasing the hardness of the active surfaces and/or of the rolling elements
Improper behavior (with irregularities, vibrations and noise)	Crushing of the materials of the surfaces in contact	In drives whose active elements are made of steels with low hardness	Increased contact stresses and improper lubrication	Squeezing out of the lubricant film; plastic deformation of the active surfaces	Limiting by calculation the contact and/or crushing pressures

of sliding screw-nut drives	(including the lubricant film)				
	Abrasive wear of the active surfaces of the thread	Drives operating in environments with dust and impurities	Exceeding the breaking limit of the material	Scratches on the active surfaces; increased clearance	Increasing the effectiveness of the sealing and lubrication systems

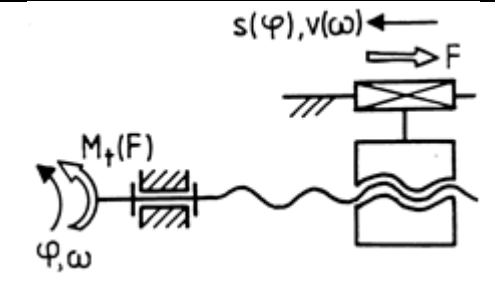
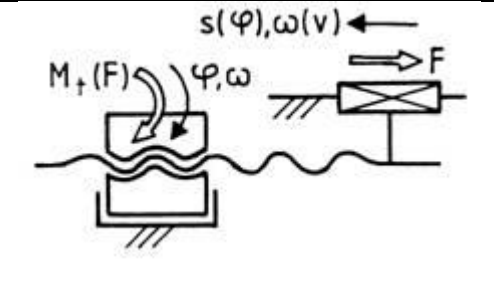
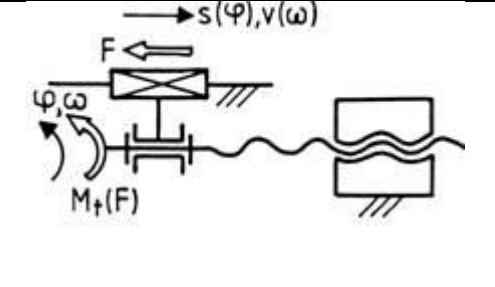
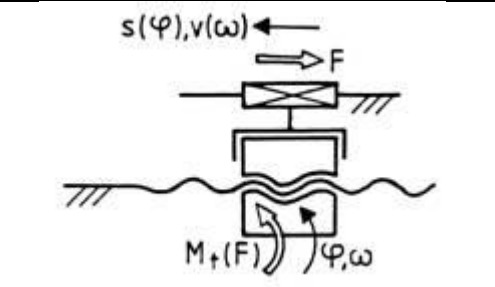
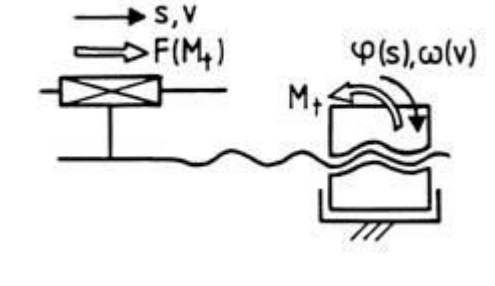
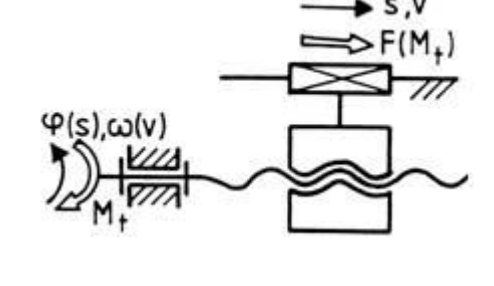
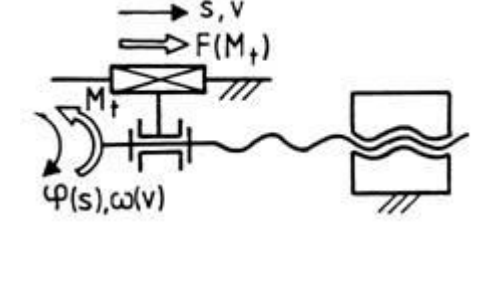
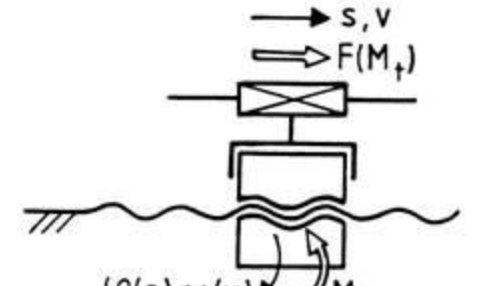
TSP-T.7 FUNCTIONAL AND CONSTRUCTIVE PARAMETERS

Helical pairs can be used in drives which, together with transforming the motion, also transmit significant loads, frequently known as actuating screw-nut drives (fig. TSP-T.2.1) or lifting screw-nut drives (jacks, subchapter TSP-T.8.1.3). When the drives with helical pairs serve only for the direct, precise transformation of motion (the transmitted loads are much smaller), they are called kinematic screw-nut drives.

Tab. TSP-T.7.1 summarizes all the possible cases of using helical pairs in screw-nut drives as mechanisms for the direct (rotation into translation) and inverse (translation into rotation) transformation of motion, with either the screw or the nut as the driving (input) element. The operation of these mechanisms depends on the values of the parameters β_2 , the helix angle of the thread, and ϕ' , the reduced friction angle.

The input/output parameters are (tab. TSP-T.7.1): $s(\varphi)$ and $v(\omega)$ – the axial displacement and velocity resulting from rotating the driving element through the angle φ and, respectively, with the angular velocity ω ; $\varphi(s)$, $\omega(v)$ – the rotation and angular velocity obtained when the driving element is displaced by the distance s and, respectively, with the input velocity v ; $M_t(F)$ – the input torque depending on the resisting force F ; $F(M_t)$ – the driving force as a function of the resisting torque M_t .

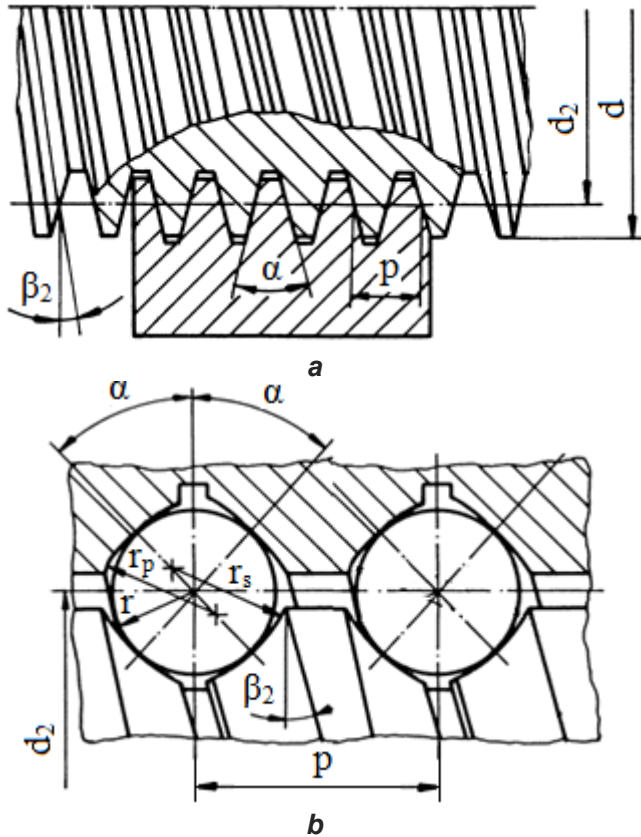
Tab. TSP-T.7.1 Types of mechanisms with helical pair (screw-nut)

$\beta_2 < \varphi'$ – the inverse transmission of motion (from output to input) is not possible (the helical pair is irreversible, locked), as a consequence of meeting the self-locking condition			
 a	 b	 c	 d
$\beta_2 > \varphi'$ – the inverse transmission of motion (from output to input) is possible (the pair is reversible), as a consequence of not meeting the self-locking condition			
 e	 f	 g	 h
Fig. TSP-T.7.1 Mechanisms with helical pair: a – with rotating, non-translating driving screw and translating nut; b – with rotating, non-translating driving nut and translating screw; c – with rotating, translating driving screw and fixed nut; d – with rotating, translating driving nut and fixed screw; e – with non-rotating, translating driving screw and rotating, non-translating nut; f – with translating driving nut and rotating, non-translating screw; g – with translating, rotating driving screw and fixed nut; h – with translating, rotating driving nut and fixed screw			

TSP-T.8 STRUCTURAL, FUNCTIONAL AND CONSTRUCTIVE MODELS

TSP-T.8.1.1 STRUCTURAL-FUNCTIONAL MODEL OF THE HELICAL PAIR

The transformation of motion in these drives is achieved by using power (motion) threads with square (fig. TSP-T.8.1.2.1,b), trapezoidal (fig. TSP-T.8.1.1.1,a; TSP-T.8.1.2.1,c), buttress or round profile. Square threads ensure maximum efficiency; their use is limited by the insufficient centering of the nut relative to the screw, by the low stiffness and strength of the thread turn and by the fact that they cannot be manufactured by milling, a high-productivity process. Trapezoidal threads ensure good centering, have higher stiffness and strength of the thread turn than the square profile and can be machined by milling.

Tab. TSP-T.8.1.1 Motion (load) transmission functions and efficiency**Fig. TSP-T.8.1.1.1** Geometry of the constant-lead helical pair: **a** – with sliding; **b** – with rolling

$s(\varphi) = \frac{p}{2\pi} \varphi$, $v(\varphi) = \frac{p}{2\pi} \omega$, $M_t(F) = \frac{d_2}{2} \operatorname{tg}(\varphi' \pm \beta_2) F$ (TSP-T.8.1.1.1)
 for the direct transformation (rotation into translation, fig. TSP-T.7.1,a,b,c,d) and

$\varphi(s) = \frac{p}{2\pi} s$, $\omega(v) = \frac{p}{2\pi} v$, $F(M_t) = \frac{1}{\frac{d_2}{2} \operatorname{tg}(\varphi' \pm \beta_2)} M_t$ (TSP-T.8.1.1.2)

for the inverse transformation (translation into rotation, fig. TSP-T.7.1,e,f,g,h). In these relations: p is the thread lead; d_2 – the mean (pitch) diameter of the thread; $\beta_2 = \operatorname{arctg}(p/\pi d_2)$, the mean helix angle of the thread; $\varphi' = \operatorname{arctg}(\mu/\cos(\alpha/2))$, the reduced friction angle; μ – the sliding friction coefficient in the thread ($\mu = 0.15 \dots 0.3$, for pairs with sliding friction; $\mu = 0.02 \dots 0.05$, for pairs with rolling friction); α – the thread profile angle ($\alpha = 0^\circ$, for the square thread; $\alpha = 30^\circ$, for the trapezoidal thread). The + sign corresponds to the case in which the direction of rotation of the driving element is the same as the direction of the thread helix, for direct (irreversible) drives (fig. TSP-T.7.1,a,b,c,d), or the direction of axial displacement of the driving element is the same as the inclination of the helix, for inverse drives (fig. TSP-T.7.1,e,f,g,h). The - sign corresponds to the case in which the direction of rotation of the driving element is opposite to the direction of the thread helix, for direct (irreversible) drives (fig. TSP-T.7.1,a,b,c,d), or the direction of axial displacement of the driving element is opposite to the inclination of the helix, for inverse drives (fig. TSP-T.7.1,e,f,g,h).

The efficiency of the helical pair, as the ratio between the useful work (for lifting the load Q by a distance equal to the lead p) and the work consumed (by rotating the screw through the angle 2π with the screwing/unscrewing torque),

$$\eta = \frac{L_u}{L_c} = \frac{Q p}{2\pi M_i} = \frac{Q p}{2\pi Q \frac{d_2}{2} \operatorname{tg}(\varphi' \pm \beta_2)} = \frac{\operatorname{tg} \beta_2}{\operatorname{tg}(\varphi' \pm \beta_2)} \quad (\text{TSP-T.8.1.1.3})$$

where β_2 is the mean helix angle of the thread and $\varphi' = \operatorname{arctg}(\mu/\cos(\alpha/2))$, the reduced friction angle. The efficiency is maximum for $\alpha = 0$ ($\varphi' = \varphi$), the case of the square power thread (fig. TSP-T.8.1.2.1,b); for other power threads $\alpha > 0$ (e.g. $\alpha = 30^\circ$, trapezoidal thread, fig. TSP-T.8.1.2.1,c), $\varphi' > \varphi$ and the efficiency is lower than that of the helical pair with square thread. When the helical pairs meet the self-locking condition ($\beta_2 < \varphi'$), at the limit ($\beta_2 = \varphi'$) it results,

$$\eta = \frac{\operatorname{tg} \varphi'}{1 - \operatorname{tg}^2 \varphi'} = \frac{1}{2} < 0.5. \quad (\text{TSP-T.8.1.1.4})$$

The maximum value of the efficiency of the helical pair, defined by relation (TSP-T.8.1.1.3), is obtained from the condition,

$$\frac{d\eta}{d\beta_2} = \frac{\sin \varphi' \cos(\varphi' + 2\beta_2)}{\cos^2 \beta_2 \cos^2(\varphi' + \beta_2)} = 0 \quad (\text{TSP-T.8.1.1.5})$$

which leads to the equation $\cos(\varphi' + 2\beta_2) = 0$, from which $\beta_2 = \pi/4 - \varphi'/2$. Substituting this parameter into relation (TSP-T.8.1.1.3), the maximum efficiency is obtained,

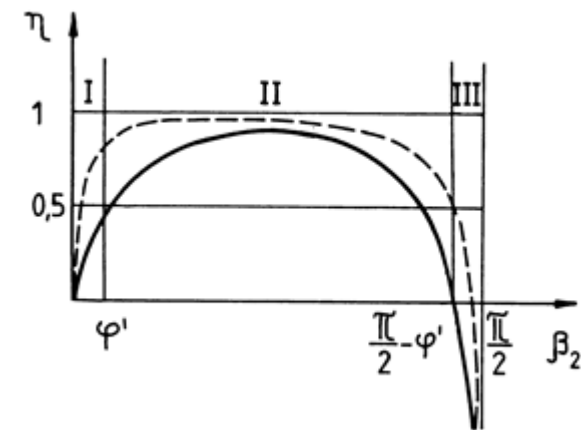
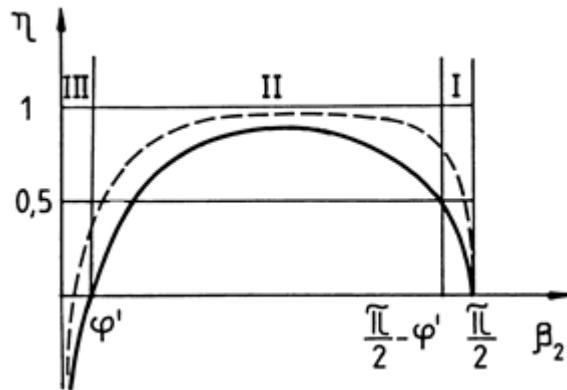
**a****b**

Fig. TSP-T.8.1.1.2 Efficiency of the helical pair: **a** – for the direct transmission (rotation into translation); **b** – for the inverse transmission (translation into rotation)

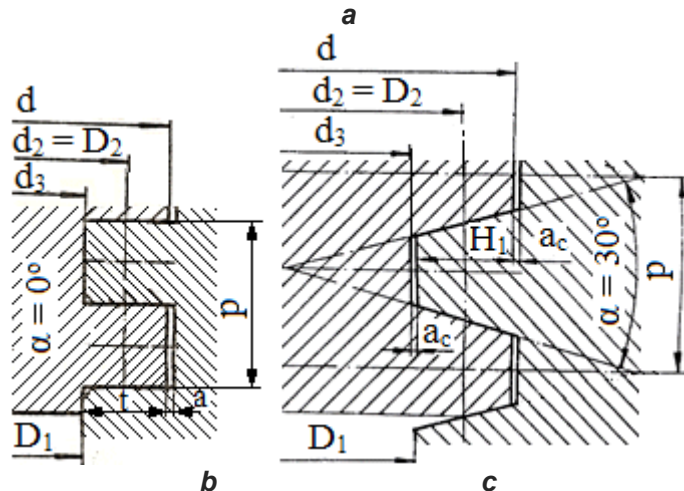
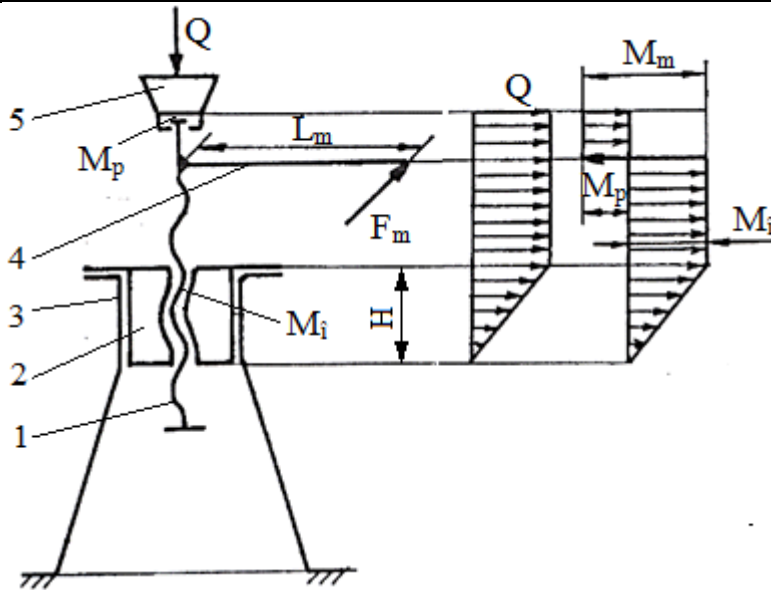
$$\eta_{\max} = \frac{\operatorname{tg}\left(\frac{\pi}{4} - \frac{\varphi'}{2}\right)}{\operatorname{tg}\left(\frac{\pi}{4} + \frac{\varphi'}{2}\right)}$$

(TSP-T.8.1.1.6)

In the operation of helical pairs, according to the graphs in fig. TSP-T.8.1.1.2, depending on the values of the parameters β_2 and φ' , the following regimes are identified: I – motion meeting the self-locking condition ($\beta_2 < \varphi'$), II – motion without meeting the self-locking condition and III – locking (continuous line for sliding helical pairs and dashed line for rolling ones).

Note. Usual measures for increasing the efficiency: using multi-start threads; reducing the coefficient of friction by using antifriction materials (e.g. bronzes for nuts); replacing sliding friction by rolling friction.

TSP-T.8.1.2 CALCULATION MODELS OF THE HELICAL PAIRS OF LIFTING DRIVES (JACKS)



Calculation assumptions:

- the load Q is lifted by driving, through crank 4, screw 1, which moves as a result of its interaction with nut 2, fixed in body 3; the moment of the force F_m acting on the screw overcomes the friction torque in the thread, M_i , and the pivot friction torque M_p (sliding between cup 5 and screw 1, or in the thrust bearing);
- uniform distribution of the force on the threads in contact.

Sizing of the helical pair

Sizing of the screw means determining the minor diameter of the thread from the compressive (tensile) strength condition,

$$d_1 = \sqrt{\frac{4 \beta Q}{\pi \sigma_{at}}}, \quad (\text{TSP-T.8.1.2.1})$$

where $\beta = 1.2 \dots 1.3$ is the coefficient that takes into account the additional torsional stress; $\sigma_{ac,t} = \sigma_{02}/c$ is the permissible compressive (tensile) stress, with $c = 2 \dots 4$ the safety coefficient; for the threads in fig. TSP-T.8.1.2.1,b,c the design parameter is d_3 instead of d_1 in relation (TSP-T.8.1.2.1).

Sizing of the nut means determining the number of threads needed from the contact pressure strength condition,

$$z = \frac{Q}{\frac{\pi}{4}(d^2 - D_1^2) p_a}, \quad (\text{TSP-T.8.1.2.2})$$

where d is the nominal diameter of the thread, D_1 – the minor diameter of the nut thread, p_a – the permissible pressure of the lubricant film ($p_a = 5 \dots 15$ MPa). The length of the nut is $H = p z$.

Verification of the screw under combined stresses (compression and torsion), according to the maximum shear stress theory, is made with the relation,

$$\sigma_{ech} = \sqrt{\sigma_c^2 + 4 \tau_t^2} \leq \sigma_{ac}, \quad (\text{TSP-T.8.1.2.3})$$

where

$$\sigma_c = \frac{4 Q}{\pi d_1^2}, \quad \tau_t = \frac{16 M_i}{\pi d_1^3}$$

are the effective compressive and, respectively, torsional stresses, $M_i = d_2 \operatorname{tg}(\varphi' + \beta_2) F/2$ being the friction torque in the thread.

The self-locking condition, from the condition of the unscrewing torque, $M_d = d_2 \operatorname{tg}(\varphi' - \beta_2) F/2 > 0$ (locking of the helical pair under load),

$$\beta_2 < \varphi', \quad (\text{TSP-T.8.1.2.4})$$

where $\beta_2 = \operatorname{arctg}(p/\pi d_2)$ is the mean helix angle of the thread and $\varphi' = \operatorname{arctg}(\mu/\cos(\alpha/2))$, the reduced friction angle.

The efficiency of the drive, considering the friction in the thread and between the bearing surfaces of the cup on the screw (fig. TSP-T.8.1.2.1,d) or in the thrust bearing (fig. TSP-T.8.1.2.1,e), is determined with the relations:

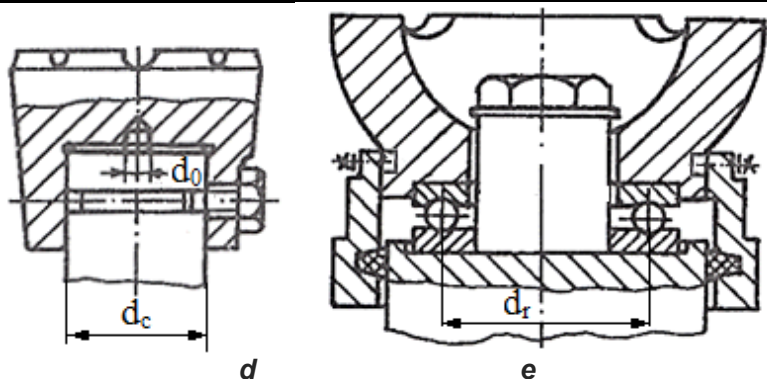


Fig. TSP-T.8.1.2.1 Jack drive with helical pair: **a** – structural scheme and load diagram; **b** – geometry of the square thread; **c** – geometry of the trapezoidal thread; **d** – construction of the cup with sliding; **e** – construction of the cup with rolling

$$\eta = \frac{L_u}{L_c} = \frac{Q p}{2\pi(M_i + M_p)} = \frac{Q \pi d_2 \operatorname{tg} \beta_2}{2\pi \left(Q \frac{d_2}{2} \operatorname{tg} (\varphi' \pm \beta_2 + \mu_1 Q \frac{d_0 + d_c}{4}) \right)} = \frac{\operatorname{tg} \beta_2}{\operatorname{tg} (\varphi' \pm \beta_2) + \mu_1 Q \frac{d_0 + d_c}{2 d_2}}, \text{ (TSP-T.8.1.2.5)}$$

or, respectively,

$$\eta = \frac{L_u}{L_c} = \frac{Q p}{2\pi(M_i + M_p)} = \frac{Q \pi d_2 \operatorname{tg} \beta_2}{2\pi \left(Q \frac{d_2}{2} \operatorname{tg} (\varphi' \pm \beta_2 + \mu_r Q \frac{d_r}{2}) \right)} = \frac{\operatorname{tg} \beta_2}{\operatorname{tg} (\varphi' \pm \beta_2) + \mu_1 Q \frac{d_r}{d_2}}. \text{ (TSP-T.8.1.2.6)}$$

TSP-T.8.1.3 CONSTRUCTIVE SOLUTIONS OF LIFTING SCREW-NUT DRIVES (JACKS)

Simple screw-nut jacks

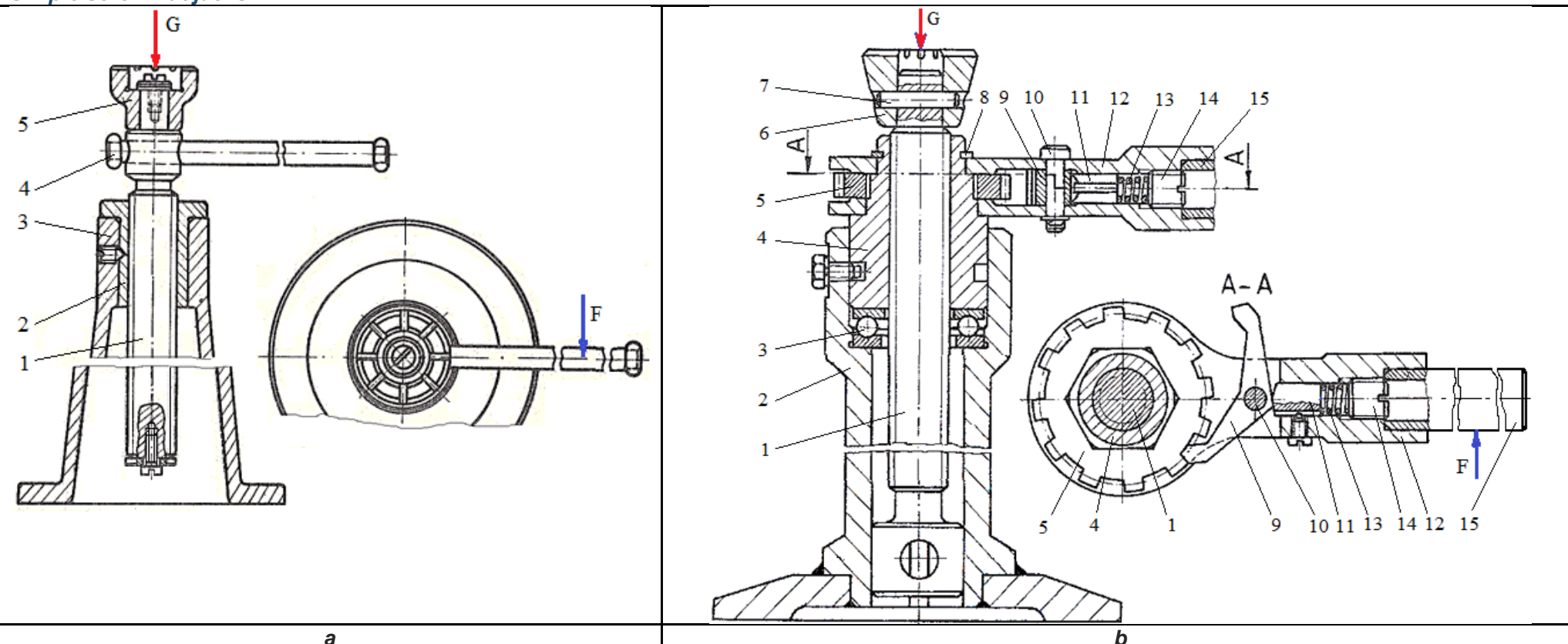


Fig. TSP-T.8.1.3.1 Simple screw-nut jacks: **a** – with rotating screw; **b** – with rotating nut

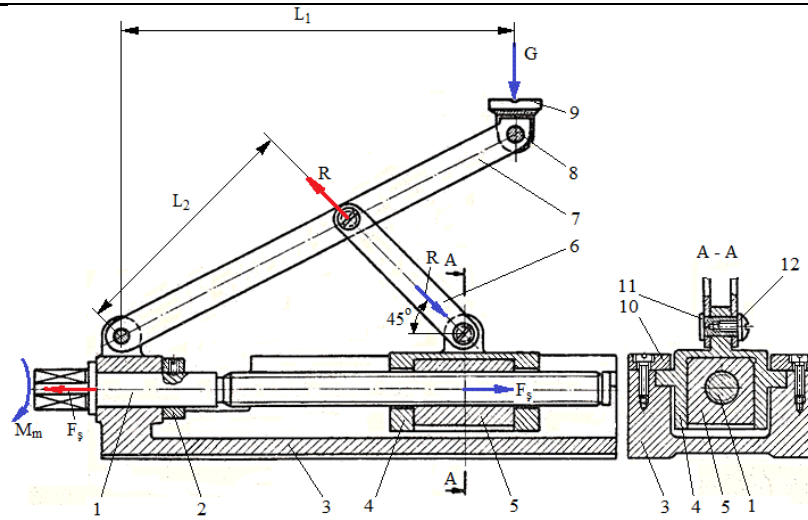
Operation and construction of the solution in fig. TSP-T.8.1.3.1,a:

The load to be lifted/pushed, the weight G , is transmitted from cup 5 through the (rotating) screw 1, the power thread (trapezoidal or square), nut 2 and body 3 to the support on which the jack is placed (metal, wood). The manual driving force F is transmitted from crank 4 to screw 1.

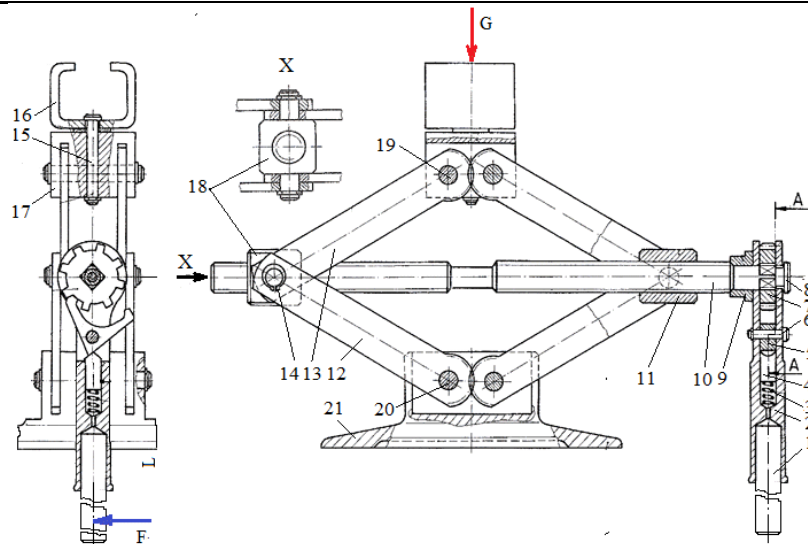
Operation and construction of the solution in fig. TSP-T.8.1.3.1,b:

The load to be lifted/pushed, the weight G , is transmitted from cup 6 through the cylindrical pin 7, the (non-rotating) screw 1, the power thread (trapezoidal or square), nut 4, the thrust ball bearing 3 and body 2 to the support on which the jack is placed (metal, wood). The manual driving force F , for lifting/pushing, is transmitted from the extension (crank) 15 through the ratchet body 12, pin 10, pawl 9 and ratchet wheel 5 (through the hexagonal-profile joint) to nut 4. To lower/retract the load G (rotating crank 15 in the opposite direction to lifting/pushing), pawl 9 is turned until the tip of its free arm enters a gap of ratchet wheel 5. To keep the pawl in position while the crank is operated, the detent 11 presses permanently on pawl 9 under the action of the helical compression spring 13, adjusted with the threaded plug 14. The circlip 8 prevents the axial displacement of the ratchet body 12.

Screw-nut jacks with lever mechanisms

**a****Operation and construction:**

The load to be lifted/pushed, the weight G , is transmitted from cup 9 through pin 8, lever 7, pin 13, bar 6, pin 11, the nut body 4, the non-rotating nut 5 and the power thread (trapezoidal or square) to screw 1 and to the frame 3. Screw 1 is driven by a crank through a square-profile joint. The nut body 4 interacts with housing 3 through the closed guideway (prismatic pair) with the guide bars 10. The washer 2, fixed with a set screw, prevents the axial displacement of screw 1 in frame 3.

**b****Operation and construction:**

The load to be lifted/pushed, the weight G , is transmitted from cup 16 through support 17, pins 19, levers 13, the nuts with pins 18/11 (one nut has a left-hand thread and the other a right-hand thread), screw 10, levers 12, pins 20 and the base plate 21 to the support on which the jack is placed (metal, wood). The manual driving force F , for lifting/pushing, is transmitted from the extension (crank) 1 through the ratchet body 2, pin 6, pawl 5, ratchet wheel 7 and the square-profile joint to screw 10. To lower/retract the load G (rotating crank 1 in the opposite direction to lifting/pushing), pawl 5 is turned until the tip of its second arm enters a gap of ratchet wheel 7. To keep the pawl in position while the crank is operated, the detent 4 presses permanently on pawl 5 under the action of the helical compression spring 3. The shoulder screw 8 and the spacer 9 prevent the axial displacement of the ratchet body 2. The circlips 14 fix the levers 13 and 12 axially. Pin 15 allows the load to be rotated/hooked on support 17.

Fig. TSP-T.8.1.3.2 Screw-nut jacks with lever mechanism: a – asymmetric; b – symmetric